

University at Buffalo
GIFTED MATH PROGRAM

Course III Curriculum

Textbooks: *The Art of Problem Solving: Introduction to Geometry* by Richard Rusczyk; *The Art of Problem Solving: Introduction to Counting and Probability* by David Patrick; Excerpts from *The Art of Problem Solving: Intermediate Algebra* by Richard Rusczyk and Matthew Crawford; and *Transformational Geometry* by Richard Brown

Unit 1: GEOMETRY

- CHAPTER 1 – WHAT’S IN A NAME?: Why Names and Symbols?; Points, Lines, and Planes; Round and Round; Construction: Copy a Segment; The Burden of Proof
- CHAPTER 2 – ANGLES: What is an Angle?; Measuring Angles; Straight and Vertical Angles; Parallel Lines; Angles in a Triangle; Exterior Angles; Parallel Lines Revisited
- CHAPTER 3 – CONGRUENT TRIANGLES: SSS Congruence; SAS Congruence; ASA and AAS Congruence; SSA Not-Necessarily Congruence; Isosceles and Equilateral Triangles; Construction: Equilateral Triangle and Perpendicular Bisector
- CHAPTER 4 – PERIMETER AND AREA: Perimeter; Area; Same Base/Same Altitude
- CHAPTER 5 – SIMILAR TRIANGLES: What is Similarity?; AA Similarity; SAS Similarity; SSS Similarity; Using Similarity in Problems; Construction: Angles and Parallels
- CHAPTER 6 – RIGHT TRIANGLES: Pythagorean Theorem; Two Special Right Triangles; Pythagorean Triples; Congruence and Similarity Revisited; Heron’s Formula; Construction: Perpendicular Lines
- CHAPTER 7 – SPECIAL PARTS OF A TRIANGLE: Bisectors; Perpendicular Bisectors of a Triangle; Angle Bisectors of a Triangle; Medians; Altitudes; Construction: Bisectors
- CHAPTER 8 – QUADRILATERALS: Quadrilateral Basics; Trapezoids; Parallelograms; Rhombi; Rectangles; Squares; If and Only If; Quadrilateral Problems
- CHAPTER 9 – POLYGONS: Introduction to Polygons; Angles in Polygons; Polygon Area; Construction: Regular Polygons
- CHAPTER 10 – GEOMETRIC INEQUALITIES: Sides and Angles of a Triangle; Pythagoras – Not just for Right Triangles?; The Triangle Inequality
- CHAPTER 11 – CIRCLES: Arc Measure, Arc Length, and Circumference; Area; Funky Areas
- CHAPTER 12 – CIRCLES AND ANGLES: Inscribed Angles; Angles Inside and Outside Circles; Tangents; Construction: Tangents
- CHAPTER 13 – POWER OF A POINT: What is Power of a Point?; Power of a Point Problems
- CHAPTER 14 – THREE-DIMENSIONAL GEOMETRY: Planes; Prisms; Pyramids; Regular Polyhedra
- CHAPTER 15 – CURVED SURFACES: Cylinders; Cones; Spheres
- CHAPTER 17 – ANALYTIC GEOMETRY: Lines; Circles; Proofs with Analytic Geometry; Distance between a Point and a Line
- CHAPTER 18 – INTRODUCTION TO TRIGONOMETRY: Trigonometry and Right Triangles; Not Just for Right Triangles; Law of Sines and Law of Cosines

Unit 2: PROBABILITY

- CHAPTER 1 – COUNTING: Counting Lists of Numbers; Counting with Addition and Subtraction; Counting Multiple Events; Permutations
- CHAPTER 2 – BASIC COUNTING TECHNIQUES: Complementary Counting; Constructive Counting; Counting with Restrictions
- CHAPTER 3 – CORRECTING FOR OVERCOUNTING: Permutations with Repeated Elements; Counting Pairs of Items; Counting with Symmetries

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- CHAPTER 4 – COMMITTEES AND COMBINATIONS: Committee Forming; How to Compute Combinations; First Combinatorial Identity
- CHAPTER 5 – MORE WITH COMBINATIONS: Paths on a Grid; Distinguishability
- CHAPTER 6 – COUNTING PROBLEMS: Harder Counting Problems
- CHAPTER 7 – INTRODUCTION TO PROBABILITY: Basic Probability; Equally Likely Outcomes; Counting Techniques in Probability Problems
- CHAPTER 8 – BASIC PROBABILITY TECHNIQUES: Probability and Addition; Complementary Probabilities; Probability and Multiplication; Probability with Dependent Events
- CHAPTER 10 – GEOMETRIC PROBABILITY: Probability Using Lengths; Probability Using Areas
- CHAPTER 11 – EXPECTED VALUE: Definition; Problems
- CHAPTER 12 – PASCAL’S TRIANGLE: Constructing Pascal’s Triangle; An Interesting Combinatorial Identity; Another Combinatorial Identity
- CHAPTER 13 – THE HOCKEY STICK IDENTITY: Problem; Solution; Identity
- CHAPTER 14 – THE BINOMIAL THEOREM: Algebra; The Binomial Theorem; Applications; Using in Identities

Unit 3: TRANSFORMATIONAL GEOMETRY

- CHAPTER 1 – THE GEOMETRY OF TRANSFORMATIONS: Maps and Mappings; One-to-One Mappings; Transformations; Mappings in Algebra; Functions; Isometries; Problems Solved by Reflections; Properties of Isometries; Rotations; Translations and Glide Reflections; Symmetry; The Fundamental Theorems of Isometries
- CHAPTER 2 – THE ALGEBRA OF TRANSFORMATIONS: The Composite (Product) of Mappings; The Algebra of Translations; The Algebra of Half-Turns; The Algebra of Rotations; Groups; Transformation Groups; Symmetry Groups

Unit 4: CONIC SECTIONS

- CIRCLES: Construction given center and radius; Equations of circles in coordinate plane
- ELLIPSES: Construction given foci-pins and string; Concentric circles construction; Paper folding construction; Equations of ellipses in coordinate plane; Eccentricity; Area; Applications of ellipses
- PARABOLAS: Construction given focus and directrix; Sliding ruler method; Concentric circles construction; Paper folding construction; Equations of parabolas in coordinate plane; application of parabolas
- HYPERBOLAS: Construction of hyperbolas based on foci; Concentric circles construction; Eccentricity; Equation of hyperbolas in coordinate place; asymptotes; application of hyperbolas
- OPTIMIZATION: Pre-calculus solutions (using ellipses) to Burning Tent and Swimming Pool problems