Guided Notes for lesson P.2 – Properties of Exponents

If $a, b, x, y \in \Re$ and $a, b, \neq 0$, and $m, n \in Z$ then the following properties hold:

1. **<u>Negative Exponent Rule:</u>** $b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$ **Answers must never contain negative exponents.**

Examples:

a) 5^{-3}	b) $\frac{1}{4^{-2}}$

2. **<u>Zero Exponent Rule:</u>** $b^0 = 1$

Examples:

a) 7^0	b) $(-5)^0$	c) -5^0

3. **<u>Product Rule:</u>** $b^m \cdot b^n = b^{m+n}$ - keep the base and ________ the exponents

Examples:

a) $2^2 \cdot 2^3$	b) $x^{-3} \cdot x^7$

4. **<u>Quotient Rule:</u>** $\frac{b^m}{b^n} = b^{m-n}$ - keep the base and ______________________ the exponents.

Examples:

a) $\frac{2^{12}}{2^4}$	b) $\frac{x^2}{x^5}$

5. **<u>Power to a Power Rule</u>** $\left(b^m\right)^n = b^{m\,n}$ - keep the base and ___________________ the exponents.

Examples:

a) $\left(2^2\right)^5$	b) $\left(x^{-3}\right)^4$

6. **<u>Product to a power:</u>** $(ab)^n = a^n b^n$ - raise both bases to the same power.

Note well: $(a+b)^n \neq a^n + b^n$ and $(a-b)^n \neq a^n - b^n$

Examples:

a) $(-2y)^4$	b) $\left(3x^{-2}y^5\right)^2$

7. **<u>Quotient to a power:</u>** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ - raise both bases to the same power.

Examples:

a) $\left(\frac{2}{5y}\right)^4$	b) $\left(\frac{x^{-3}}{3y^{-2}}\right)^2$

Simplifying Exponential Expressions

1) No parentheses.
2) No powers raised to a powers.
3) Each based occurs only once.
4) No negative exponents.
5) Simplify numerical expressions.

Examples: Simplify the expression.

a) $\left(2x^8y^{-4}\right)^2$	b) $\left(-3x^4y^5\right)^3$
c) $\left(-7xy^4\right)\left(-2x^5y^6\right)$	d) $\left(-6x^2y^5\right)\left(3xy^{-3}\right)$
e) $\dfrac{-35x^2y^4}{5x^6y^{-8}}$	f) $\dfrac{100x^{-2}y^6}{20x^{-4}y^3}$
g) $\left(\dfrac{4x^2d^3}{y^{-7}g^4}\right)^{-3}$	h) $\left(\dfrac{5x^{-8}}{2y^4}\right)^{-5}$

Definition: A number is written in ______________________________ if it is of the form _____________ where $1 \le |a| < 10$ and $n \in Z$.

Examples : Write in standard form (decimal form).

a) 6.4×10^5	b) -1.25×10^7
c) 2.17×10^{-3}	d) -6.018×10^{-5}

Example 2: Write in scientific notation.

a) 34,970,000,000,000	b) –0.000000000000802

To multiply numbers written in scientific notation:
1. Multiply the decimal parts first. (use your calculator, if necessary)
2. Using the rules of exponents, multiply the powers of ten. (add the exponents)
3. Be sure your final answer is in scientific notation.

$$\left(2 \times 10^6\right) \times \left(4 \times 10^7\right)$$
$$\left(2 \times 4\right) \times \left(10^6 \times 10^7\right)$$
$$8 \times 10^{13}$$

a) $\left(3 \times 10^5\right) \times \left(2 \times 10^7\right)$	b) $\left(5 \times 10^3\right) \times \left(3 \times 10^5\right)$	c) $\left(6 \times 10^2\right) \times \left(7 \times 10^8\right)$
d) $\left(8 \times 10^{-3}\right) \times \left(4 \times 10^{-5}\right)$	e) $\left(9 \times 10^8\right) \times \left(3 \times 10^{-7}\right)$	f) $\left(6 \times 10^{-6}\right) \times \left(2.3 \times 10^{-4}\right)$

To divide numbers written in scientific notation: 1. Divide the decimal parts first. (use your calculator, if necessary) 2. Using the rules of exponents, divide the powers of ten. (subtract the exponents) 3. Be sure your final answer is in scientific notation.	$\left(4 \times 10^{15}\right) \div \left(2 \times 10^{7}\right)$ $\left(4 \div 2\right) \times \left(10^{15} \times 10^{7}\right)$ 2×10^{8}

Examples: Find the quotient, express answers in scientific notation.

a) $\dfrac{8 \times 10^{5}}{2 \times 10^{3}}$	b) $\dfrac{6 \times 10^{12}}{3 \times 10^{8}}$	c) $\dfrac{9 \times 10^{4}}{3 \times 10^{7}}$
d) $\dfrac{8.24 \times 10^{-24}}{5.15 \times 10^{-18}}$	e) $\dfrac{3 \times 10^{-5}}{6 \times 10^{8}}$	f) $\dfrac{2 \times 10^{3}}{5 \times 10^{-12}}$

Guided Notes for lesson P.3 – More with Exponents

If $b \in \Re$ and $b \neq 0$, and $m, n \in Z$ then the following properties hold:

8. **Fractional Exponents:** $b^{\frac{1}{n}} = \sqrt[n]{b} \begin{cases} \text{If } n \text{ is even, } b \geq 0 \\ \text{If } n \text{ is odd, } b \in \Re \end{cases}$ and $b^{\frac{-1}{n}} = \frac{1}{\sqrt[n]{b}} \begin{cases} \text{If } n \text{ is even, } b > 0 \\ \text{If } n \text{ is odd, } b \in \Re \end{cases}$

Examples:

a) $64^{\frac{1}{2}}$	b) $125^{\frac{1}{3}}$	c) $-16^{\frac{1}{4}}$
d) $(-27)^{\frac{1}{3}}$		e) $64^{\frac{-1}{3}}$

9. **Fractional Exponents:** $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m \begin{cases} \text{If } n \text{ is even, } b > 0 \\ \text{If } n \text{ is odd, } b \in \Re \end{cases}$ and $b^{\frac{-m}{n}} = \frac{1}{\sqrt[n]{b^m}} = \frac{1}{\left(\sqrt[n]{b}\right)^m} \begin{cases} \text{If } n \text{ is even, } b > 0 \\ \text{If } n \text{ is odd, } b \in \Re \end{cases}$

Examples:

a) $27^{\frac{2}{3}}$	b) $4^{\frac{3}{2}}$	c) $8^{\frac{5}{3}}$
d) $81^{\frac{-3}{4}}$		e) $(-1000)^{\frac{-4}{3}}$

Examples: Simplify the expression. Express your answer using a radical when your final answer yields a fractional exponent.

a) $\left(5x^{\frac{1}{2}}\right)\left(7x^{\frac{3}{4}}\right)$	b) $\left(2x^{\frac{5}{3}}y^{\frac{1}{6}}\right)^{6}$
c) $\dfrac{32x^{\frac{7}{3}}}{16x^{\frac{1}{4}}}$	d) $\left(\dfrac{x^{\frac{1}{4}}y^{\frac{-2}{3}}}{x^{\frac{3}{2}}y^{\frac{7}{4}}}\right)^{12}$
e) $\left(\sqrt{x}\right)\left(\sqrt[3]{x}\right)$	f) $\left(\sqrt[4]{x^5}\right)\left(\sqrt[3]{x^7}\right)$

Examples: Factor the expression and simplify. This is a skill in preparation for calculus.

a) $(x+1)^{\frac{3}{2}} + \frac{3x}{2}(x+1)^{\frac{1}{2}}$	b) $\left(x^2+4\right)^{\frac{4}{3}} + \frac{8x}{3}\left(x^2+4\right)^{\frac{1}{3}}$
c) $-2x\left(8-x^2\right)^{\frac{-1}{2}} + \left(8-x^2\right)^{\frac{1}{2}}$	

Guided Notes for lesson P.7 – Absolute Value Equations

Absolute Value:

Definition: $\|a\|$ means the distance the number a is from ________ on a number line.

Solving Absolute Value Equations:

If X is any algebraic expression and $c \in Z^{+}$, then the solutions to $\|X\| = c$ are found by solving the equations ___________ and ___________.

Example Solve the equation.

a) $\|x\| = 5$	b) $\|x - 3\| = 7$
c) $\|x + 8\| = 4$	d) $\|2x - 3\| = 15$
e) $\left\|\frac{1}{3}x + 7\right\| = 4$	f) $\|4 - 5x\| = 7$

Guided Notes for lesson P.9 – Inequalities and Absolute Value Equations and Inequalities

To solve an inequality means to find all the possible values of the variable that makes the inequality true. To solve an inequality you use the same inverse operation(s) to both sides of the inequality with one slight glitch. Keep in mind you want the variable all alone on one side of the inequality.

When an inequality is multiplied or divided by a **positive** number the inequality sign remains unchanged.
Example to illustrate:

$5 < 6$ true

$2(5) < 2(6)$ multiply by 2

$10 < 12$ true

$50 > 35$ true

$\frac{50}{5} < \frac{35}{5}$ divide by 5

$10 < 7$ true

When an inequality is multiplied or divided by a **negative** number the inequality sign is **reversed**.
Example to illustrate:

$5 < 6$ true

$-2(5) < -2(6)$ multiply by -2

$-10 < -12$ false

$-10 > -12$ true

$50 > 35$ true

$\frac{50}{-5} > \frac{35}{-5}$ divide by -5

$-10 > -7$ false

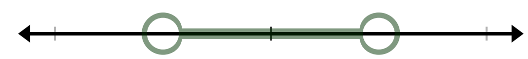
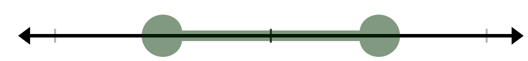
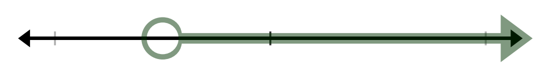
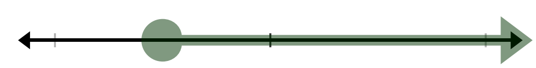
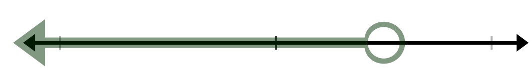
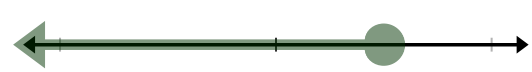
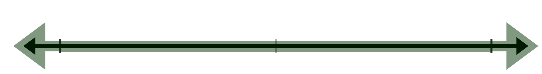
$-10 < -7$ true

Examples: Solve each of the following and graph your solution on a number line.

a) $x - 5 \geq 2$	b) $-3 \geq x + 5$
c) $4n < 16$	d) $\frac{n}{2} > -8$

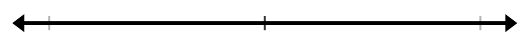
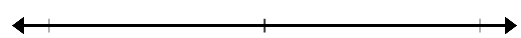
e) $\frac{x}{-10} \le 22$	f) $-13x \ge -208$
g) $-6x - 15 > 57$	h) $-4x - 40 \ge -7x + 65$
i) $12(2x - 13) < 117 - 15x$	
j) $-3 \le 5x + 2 \le 9$	i) $-2 \le 1 - 3x \le 8$

Three ways to display solutions to inequalities:

Set builder Notation (used for Alg 2)	Interval Notation (used in pre-calc on up)	Graph (used for either Alg 2 or pre-calc on up)
$x \mid a < x < b$		
$x \mid a \le x \le b$		
$x \mid x > a$		
$x \mid x \ge a$		
$x \mid x < b$		
$x \mid x \le b$		
$x \mid x \in \Re$		

Connectors of sets of numbers:

	Set builder Notation Symbol	Interval Notation Symbol
Intersection (and) **(what is in both)**		
Union (or) **(what is in either or both)**		

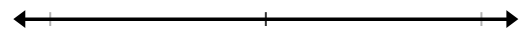
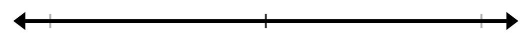
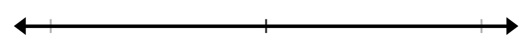
Set builder Notation	Interval Notation	Graph
$x \mid 1 < x < 4 \wedge 2 \le x \le 8$		
$x \mid 1 < x \le 2 \vee 7 \le x < 9$		

Solving Absolute Value Inequalities

Solution: If X is any algebraic expression and $c \in Z^{+}$, then the solutions to $|X| < c$ are found by solving

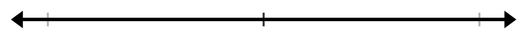
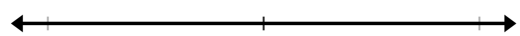
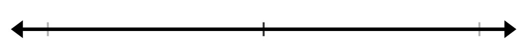
__________ and __________.

Examples: Solve the inequality and graph the solution set.

a) $\|x-5\| < 4$	
b) $\|2x+3\| \le 1$	
c) $2\|5-3x\| - 7 \le 13$	

Solution: If X is any algebraic expression and $c \in Z^{+}$, then the solutions to $|X| < c$ are found by solving ________________ and ________________.

Examples: Solve the inequality and graph the solution set.

a) $\lvert x+7 \rvert > 10$	
b) $\left\lvert \frac{1}{4}x - 3 \right\rvert \geq 8$	
c) $\lvert 5-2x \rvert > 7$	

Guided Notes for lesson P.3A – Radicals **Date: _________**

Definition: $\sqrt[n]{a}$ is called the ______________________ nth root or ______________________.

a is called the _________________________ and n is called the _________________________**.**

Note Well: If n is even, $a \geq 0$. If n is odd, $a \in \Re$.

If $a, b \in \Re$ and $b \neq 0$, and $m, n \in Z$ then the following properties hold:

1. **<u>Product Rule:</u>** $\sqrt[n]{ab} = \sqrt[n]{a} g \sqrt[n]{b}$. **Note Well** $\sqrt[n]{(a+b)} \neq \sqrt[n]{a} + \sqrt[n]{b}$ and $\sqrt[n]{(a-b)} \neq \sqrt[n]{a} - \sqrt[n]{b}$

2. **<u>Quotient Rule:</u>** $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3. **<u>Power Rule 1:</u>** $\sqrt[n]{a^n} = \begin{cases} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$

4. **<u>Power Rule 2:</u>** $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

To simplify $\sqrt[n]{a^m}$ ____

____

____

Example: Simplify the radical. Express the answer in simplest radical form.

a) $\sqrt{32}$	b) $\sqrt{75}$	c) $\sqrt{700}$
d) $\sqrt[3]{48}$	e) $\sqrt[3]{375}$	f) $\sqrt[3]{686}$
g) $\sqrt[3]{-250x^4}$	h) $\sqrt{50x^5y^6}$	i) $\sqrt[5]{64x^9y^{12}}$
j) $\sqrt{\dfrac{49x^5}{64y^9}}$		k) $\sqrt[3]{\dfrac{-54x^6}{8y^7}}$

Definition: Like radicals are radicals with the same ______________________ and ____________________

Rule: You may only add or subtract like radicals.

Example: Simplify the expression.

a) $8\sqrt{3} + 5\sqrt{3}$	b) $7\sqrt[3]{5} - 11\sqrt[3]{5}$
c) $-8\sqrt{12} + \sqrt{3}$	d) $\sqrt[3]{8x^4} + \sqrt[3]{-x} + 4\sqrt[3]{27x}$

Example: Multiply. Express the answer in simplest radical form, if necessary.

a) $8\sqrt{3} \cdot \frac{1}{2}\sqrt{15}$	b) $3\sqrt{2}\left(2\sqrt{8} - \sqrt{3}\right)$
c) $\left(2 + \sqrt{5}\right)\left(3 + \sqrt{5}\right)$	d) $\left(4 + \sqrt{7}\right)\left(4 - \sqrt{7}\right)$

Definition: The irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are called ______________________________ and when multiplied together will always yield a _________________________________ number. Why?

e) $\left(10-\sqrt{2}\right)^2$	f) $\left(2-\sqrt{3}\right)^3$

Example: Divide. Express the answer in simplest radical form, if necessary.

a) $\dfrac{48\sqrt{54}}{12\sqrt{3}}$	b) $\dfrac{10\sqrt[3]{48}}{5\sqrt[3]{2}}$
c) $\dfrac{4\sqrt{45}+6\sqrt{60}}{2\sqrt{5}}$	d) $\dfrac{\sqrt{15}-\sqrt{180}}{\sqrt{5}}$

Definition: To ____ means to find an equivalent fraction with a denominator that is a rational number. Why was/is that important?

$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{2}}{2}$

Example: Rationalize the denominator.

a) $\dfrac{2}{\sqrt{5}}$	b) $\dfrac{10}{\sqrt[3]{3}}$
c) $\dfrac{2}{4+\sqrt{11}}$	d) $\dfrac{\sqrt{3}}{\sqrt{3}+1}$
e) $\dfrac{-3+\sqrt{10}}{2+\sqrt{10}}$	f) $\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

Guided Notes for lesson 10.5 – The Binomial Theorem

Definition: A ________________ is a two termed algebraic expression.

Definition: When any binomial is raised to a positive integral power, the result is called an _____________

Illustration: Expand $(x+y)^3$

$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y)\\ &= (x+y)\left(x^2+2x^1y^1+y^2\right)\\ &= x^3+2x^2y^1+xy^2+x^2y^1+2xy^2+y^3\\ &= x^3+3x^2y^1+3x^1y^2+y^3\end{aligned}$$

A few things you should notice in the **expansion of** $(x+y)^3$:

1) the x's decrease in power $\left(x^3, x^2, x^1, x^0\right)$ term by term.
2) the y's increase in power $\left(y^0, y^1, y^2, y^3\right)$ term by term.
3) the exponents on x and y always add up to 3 for each term.
4) the number of terms (4) is one greater than the exponent 3.
5) there are coefficients on the two middle terms

Where the coefficients of a binomial expansion come from?

Definition: The coefficient of any term of binomial expansion is called a binomial coefficient and is found by $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ provided $n, r \in$ ¢$^{+}$ and $n \geq r$.

${}_nC_r$ is used as well to denote $\binom{n}{r}$. A combination of n things taken r at a time.

Examples: Evaluate by hand

a) $\binom{3}{2}$	b) $\binom{8}{5}$	c) $\binom{7}{1}$

Example: Evaluate with your calculator

a) $\binom{9}{9}$	b) $\binom{12}{7}$	c) $\binom{5}{0}$

The Binomial Theorem: For any monomial expression *a* any monomial expression *b*, and $n, r \in$ ¢$^{+}$:

$$(a+b)^n = \binom{n}{0}(a)^n(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \binom{n}{2}(a)^{n-2}(b)^2 + \ldots + \binom{n}{n}(a)^2(b)^{n-2} + \binom{n}{n}(a)^1(b)^{n-1} + \binom{n}{n}(a)^0(b)^n$$

Examples: Expand the expression. (write small)

a) $(x+3)^4$

b) $(x-2)^3$ remember that $(x-2)^3 = (x+(-2))^3$

c) $(2x+y)^5$ make sure it's the 2 and the *x* that are raised to the exponents.

d) $(3x-4y)^3$
e) $(x^2+6y^3)^4$

Consider $(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

Another way to aid you with expansion of $(x+y)^3$ is to use Pascal's Triangle

1

1 1

1 2 1

1 3 3 1 $\rightarrow x^3+3x^2y+3xy^2+y^3$

Example: Expand $(x+3y)^5$ using Pascal's Triangle.

Consider the expansion of $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

If we just wanted the **second term** of its expansion without expanding it, how could we find it?

It would be $\binom{3}{1}(x)^{3-1}(y)^1 = 3x^2y$

> For any monomial expression *a* any monomial expression *b*, and $n, r \in ¢^+$ **The rth term of a Binomial Expansion without expanding is:**
>
> $$\binom{n}{r-1}(a)^{n-r+1}(b)^{r-1}$$

Examples: Find the given term of the epansion with out expanding it.

a) $(x+2)^5$ (third term)	b) $(x-6)^8$ (fourth term)
c) $(3x+5)^6$ (sixth term)	d) $(2x-7)^9$ (second term)
e) $(x^3+2y^5)^4$ (third)	

Good luck to: ______________________________

GMP 4: Chapter PA Test: Sections: 10.5, P.2, P.3, and P.9

Multiple Choice Questions: For 1-18, circle the best answer to the question. When you see the saw graphic, SHOW ALL WORK in the space provided.

- **Correct answers with no work will receive minimal credit.**
- **Incorrect answers with work will receive partial credit.**
- **Incorrect answers with no work will receive no credit.**

You may use a calculator. Each question is worth 9 points.

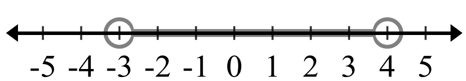
1) What is the solution of the inequality $|x+3| \le 5$?

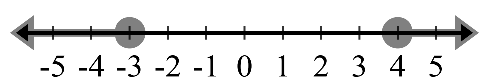
a) $\{x \mid -8 \le x \le 2\}$ b) $\{x \mid -2 \le x \le 8\}$ c) $\{x \mid x \le -8 \text{ or } x \ge 2\}$ d) $\{x \mid x \le -2 \text{ or } x \ge 8\}$

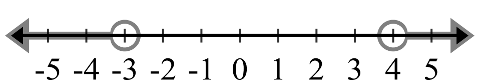
2) What is the solution of the inequality $|2x-3| > 4$?

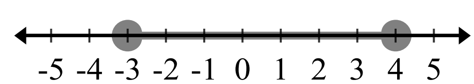
a) $\left\{x \,\middle|\, \frac{7}{2} < x < -\frac{1}{2}\right\}$ b) $\left\{x \,\middle|\, -\frac{1}{2} < x < \frac{7}{2}\right\}$ c) $\left\{x \,\middle|\, x < \frac{-1}{2} \text{ or } x > \frac{7}{2}\right\}$ d) $\left\{x \,\middle|\, x < \frac{7}{2} \text{ or } x > -\frac{1}{2}\right\}$

3) Which graph represents the solution to the inequality $|1-2x| < 7$?

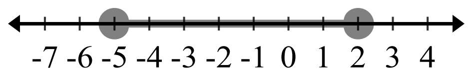
a) 

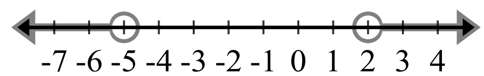
b) 

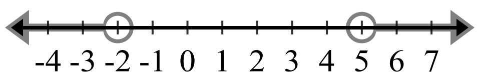
c) 

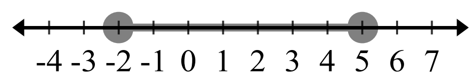
d) 

4) Which graph represents the solution to the inequality $|2x-3|>7$?

a) 

b) 

c) 

d) 

5) Evaluate the binomial coefficient: $\binom{10}{5}$.

a) 120 b) 252 c) 30,240 d) 3,628,800

6) Evaluate the binomial coefficient: $\binom{4}{1}$.

a) 24 b) 4 c) 2 d) 1

7) Expand $(5x+2)^3$ using the binomial theorem.

a) $125x^3+8$

b) $125x^3+150x^2+16x+8$

c) $125x^3+150x^2+60x+8$

d) $125x^3+30x^2+30x+8$

8) Expand $\left(x^2 - 2y\right)^4$ using the binomial theorem.

a) $x^8 - 8x^6y + 24x^4y^4 + 8x^2y^3 + 16y^4$

b) $x^8 - 2x^6y + 24x^4y^2 - 16x^2y^3 + 16y^4$

c) $x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$

d) $x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

9) The **fifth term** in the expansion of $\left(2x - y\right)^6$ is

a) $-240x^2y^4$ b) $240x^2y^4$ c) $-60x^2y^4$ d) $60x^2y^4$

9) The last term in the expansion of $\left(x + 3y\right)^4$ is

a) $108xy^3$ b) $81y^4$ c) $3y^4$ d) $243y^5$

10) The middle term in the expansion of $\left(x - y\right)^4$ is

a) $-4x^3y$ b) $-4xy^3$ c) $6x^2y^2$ d) $-6x^2y^2$

11) Simplify $\sqrt{500}$

a) $50\sqrt{10}$ b) $10\sqrt{5}$ c) $5\sqrt{10}$ d) $10\sqrt{50}$

12) Simplify $\sqrt{30}g\sqrt{12}$

a) $9\sqrt{20}$ b) $12\sqrt{10}$ c) $6\sqrt{10}$ d) $3\sqrt{40}$

13) Simplify $5\sqrt[3]{16} + 2\sqrt[3]{54}$

a) $16\sqrt{3}$ b) $16\sqrt[3]{2}$ c) $20 + 6\sqrt[3]{6}$ d) $16\sqrt[3]{4}$

14) Simplify $\sqrt{75a^9b^2c^5}$

a) $15a^4bc^2\sqrt{5a^5c^3}$ b) $3a^4bc^2\sqrt{5ac}$ c) $5a^4bc^2\sqrt{3ac}$ d) $5a^7c^3\sqrt{3ac}$

15) Simplify $\left(5+\sqrt{2}\right)\left(3\sqrt{2}-4\right)$

a) $11\sqrt{2}-14$ b) $11\sqrt{2}-8$ c) $14\sqrt{2}-20$ d) $4\sqrt{2}-14$

16) Simplify $\frac{\sqrt{128}-\sqrt{72}}{\sqrt{8}}$

a) 1 b) 2 c) $2+\sqrt{3}$ d) $5+\sqrt{3}$

17) Simplify $\frac{12}{3+\sqrt{3}}$

a) $12-\sqrt{3}$ b) $2+\sqrt{3}$ c) $4-2\sqrt{3}$ d) $6-2\sqrt{3}$

18) A Space station rotates in order to simulate gravity, such that $N=\frac{142\sqrt{15}}{\pi\sqrt{7r}}$, where N is the number of rotations per minute required to simulate earth's gravity, and r is the radius of the space station. If the number of rotations per minute is 10, which of the following is a reasonable estimate for the radius of the space station?

a) 135m b) 5.4m c) 10m d) 44m

Free Response Questions: For 19-43. When you see the saw graphic, SHOW ALL WORK in the space provided.

- **Correct answers with no work will receive minimal credit.**
- **Incorrect answers with work will receive partial credit.**
- **Incorrect answers with no work will receive no credit.**

You may use a calculator. Each question is worth 7.5 points.

For 19-22, simplify the expression completely with only positive exponents.

19) $\left(-2x^6y\right)\left(-6x^2y^5\right)$

19) ______________________

20) $\dfrac{30x^3y^{11}z^6}{5x^5y^8z^5}$

20) ______________________

21) $\left(4x^{-2}y^8z^9\right)^{-4}$

21) ______________________

22) $\left(\dfrac{-12x^5y^6}{6x^{11}y^{-2}}\right)^3$

22) ______________________

23) Write 4.56×10^{-6} in standard form without the use of exponents.

23) ______________________

24) Write 6,400,000 in scientific notation.

24) ______________________

25) Evaluate $\dfrac{\left(6.0\times10^{3}\right)\left(2.0\times10^{-7}\right)}{\left(4.0\times10^{-5}\right)\left(8.0\times10^{2}\right)}$ (express answer in scientific notation)

25) ____________________

For 26-28, simplify and express answer in <u>radical form</u>, if necessary.

26) $\left(10x^{\frac{1}{3}}\right)\left(5x^{\frac{1}{2}}\right)$ 26) ____________________	27) $\dfrac{72x^{\frac{1}{4}}}{8x^{\frac{1}{5}}}$ 27) ____________________
28) $\left(81x^{4}y^{-10}\right)^{\frac{-1}{2}}$ 28) ____________________	

For 29-34, solve the inequality. Graph your solution on a number line.

29) $n + 24 \geq 9$	30) $x - 14 < 54$
31) $-2.6y > 1.3$	32) $-8 \geq \frac{k}{4}$
33) $-4p - 1 \leq -9$	34) $2d - 10 > 6(7 - 4d)$

For 35-36, solve algebraically for *x*.

35) $\lvert x+6\rvert = 8$ 35) ______________________	36) $2\lvert 5x-1\rvert = 10$ 36) ______________________

For 37-38, solve algebraically for *x*. Express your answer in interval notation.

37) $\lvert x-7\rvert - 1 \leq 4$ 37) ______________________	38) $\lvert 3-7x\rvert > 9$ 38) ______________________

For 39-40, expand the expression completely.

39) $(3x + 5y)^4$

39) ______________________________

40) $(x^3 - y^2)^3$

40) ______________________________

41) Express in simplest radical form.

a) $\sqrt{48x^5y^9}$ a) ______________________________	b) $\sqrt[3]{40x^7y^8}$ b) ______________________________

42) Simplify the expression.

a) $2\sqrt{45} + 3\sqrt{125} - \sqrt{405}$ a) ______________________	b) $\left(6 - \sqrt{11}\right)\left(9 - \sqrt{11}\right)$ b) ______________________
c) $\left(2 + \sqrt{5}\right)^2$ c) ______________________	d) $\dfrac{3 - \sqrt{5}}{2 + \sqrt{5}}$ d) ______________________

Bonus) Find the exact value of: $\sqrt{13 + \sqrt{2} + \frac{7}{3 + \sqrt{2}}}$ (+5)

Bonus) ____________________