Name: ____

PROBABILITY UNIT & STATISTICS UNIT
GMP IV Summer Assignment 2024
Submit on Brightspace on or before August 16, 2024

The summer assignment will be calculated in your first quarter grades and includes 17 lessons, 17 homework assignments, 6 activities, 2 quizzes, and 2 tests. This is not meant to be done in one sitting nor haphazardly at the last minute. You should plan ahead and pace yourself so that you are doing one lesson at a time. You will get much more out of it this way. Please take notice of any comments that I have made for each assignment, activity, quiz, or test.

You are expected to watch the video for each lesson, complete the notes for each lesson, and complete the assignment for each lesson. You will be submitting the lesson, assignment, any graphs you make, and any data you collect from the simulation programs on or before August 16, 2024. Neatly show all work in the space provided and answer in complete sentences to get full credit. Clearly print your name in the "Name" section of each page.

For lessons 13.5, 13.6, and 13.7 you are required to use a simulation program. These can be found at: https://www.emathinstruction.com/courses/common-core-algebra-ii/statistical-simulators/.
You should print a copy of your data, put your name on it, and submit it with your work.

Use the checklist below to assist you and if you have any questions my email address is pshepard@buffalo.edu. I will be out of the country from July 10th – August 4th.

UNIT 12 PROBABILITY CHECKLIST

Lesson 12.1 Introduction to Probability

Assignment Comments:
For all problems, express all probabilities as fractions in simplest form.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 12.2 Sets and Probability

Assignment Comments:
For all problems, express all probabilities as fractions in simplest form.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 12.3 Adding Probabilities

Assignment Comments:
For all problems, express all probabilities as two place decimals

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 12.4 Conditional Probability

Assignment Comments:
For all problems, express all probabilities as two place decimals.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Additional Practice with Conditional Probability Activity

Activity Comments:
For #1, express your answers as one place percents.
For #2, express your answers as one place percents.
For #3, express your answers as one place percents.
For #4, express your answers as three place decimals.
For #5, express your answers as a fractions in simplest form.
For #6. express your answers as two place decimals.
For #7, express your answers as fractions in simplest form.

			Activity Completed		

Unit 12 Quiz

Quiz Comments:
For #7, express answers as a faction in simplest form.
For #8a, express your answer as a fraction in simplest form.
For 8b, express your answer as a three placed decimal.

			Quiz Completed		

Lesson 12.5 Independent Events

Assignment Comments:
For #2, express your answer as percents.
For #3, express your answers as fraction in simplest form.
For #5, express your answer as a as a one placed percent.
For #6 and 7 express your answer as a two placed decimal.

	Video Watched		**Lesson Completed**		**Assignment Completed**

I know the probability you just read this is 1.

More Work with Independence Activity

Assignment Comment:
For #1, #3, and #7 express your answer as a two placed decimal.
For #2 and #4, express your answer as a faction in simplest form.
For #6, express your answers as a three placed decimal.
For #8, express your answer as a whole number percent.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 12.6 Multiplying Probabilities

Assignment Comment:
For #1 and #3, express your answers as a faction in simplest form.
For #4, express your answers as a three placed decimal.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Practice Using Products to Calculate Probabilities Activity

Assignment Comment:
For #2, #4, and #6 express your answers as a faction in simplest form.
For #5 and #7 express your answers as a decimal.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Unit 12 Test

Test Comments:
For #8 and #9, express your answers as a fraction in simplest form.
For #10, express your answer as a whole number percent.
For #11, express your answer as a three placed decimal.

			Test Completed		

"

"How do you want it - crystal ball mumbo-jumbo or theoretical probability?"

"I wish we hadn't learned about probability 'cause I don't think our odds are good."

UNIT 13 STATISTICS CHECKLIST

Lesson 13.1 Variability and Sampling

Assignment Comments:
None

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 13.2 Population Parameters

Assignment Comments:
For #6a and #6b, express your answers to the nearest tenth.
For #6c, express your answers to the nearest whole number percent.
For #7a, express your answer to the nearest tenth.
For #7b express your answers to the nearest hundredth.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 13.3 The Normal Distribution

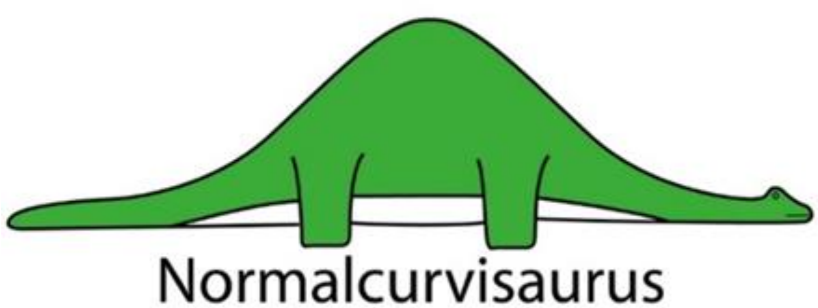
Assignment Comments:
For #8e, express your answers to the nearest tenth of a percent.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Normal Distributions Practice Activity

Activity Comments:
For #2a, #2b, and #2c, express your answers to the nearest thousandth.
For #3a, express your answers to the nearest tenth of a percent.
For #3b, express your answers to the nearest whole number percent.
For #5, express your answers to the nearest tenth.

	Video Watched		**Lesson Completed**		**Assignment Completed**



Lesson 13.4 The Normal Distribution and Z-Scores

Assignment Comments
For #6, #7a, and #7b, express your answer to the nearest whole number percent.
For #7c, express your answer to the nearest tenth of a percent.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 13.4.5 Sampling a Population

Assignment Comments:
Use the attached data for Exercise #6.

			Assignment Completed		

Lesson 13.5 Sample Means

Assignment Comments:
You will need to run the PSIMUL simulation program, print a copy of your data, and submit it with the rest of your work.
For #1a, express your answer to the nearest hundredth.
For #5, express your answer to the nearest tenth.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Lesson 13.6 Sample Proportions

Assignment Comments:
You will need to run the NORMSAMP simulation program, print a copy of your data, and submit it with the rest of your work.
For #12d, express your answers to the nearest tenth.

	Video Watched		**Lesson Completed**		**Assignment Completed**

Unit 13 Quiz

Quiz Comments:
For #10, express your answers as a two placed decimal.

			Quiz Completed		

Birthdays
are good for you.
Statistics show that
people who have the
most live the longest!
(Larry Lorenzoni)

Lesson 13.7 The Difference in Samples Means Assignment Comments: You will need to run the MEANCOMP simulation program, print a copy of your data, and submit it with the rest of your work. For #6, if the histogram that you are ask to make for does not fit, adjust the scale.					
	Video Watched		Lesson Completed		Assignment Completed

Lesson 13.8 The Distribution of Sample Means Assignment Comments: For #4a, express your answers to the nearest whole number percent, For #4b and #4c, express your answers to the nearest tenth of a percent. For 5a, express your answers for the $\sigma_{\bar{x}}$ and express your answers for the probability to the nearest tenth of a percent. For #6, express your answers to the nearest whole number.					
	Video Watched		Lesson Completed		Assignment Completed

Lesson 13.9 The Distribution of Sample Proportions Assignment Comments: For #1, express your answers to the nearest thousandth, For #2, #4, and #6 express your answers for the σ_p to the nearest thousandth and express your answers for the probability to the nearest tenth of a percent.					
	Video Watched		Lesson Completed		Assignment Completed

Statistical Simulation Packet Activity Activity Comments: For #4, express your answers to the nearest hundredth.					
			Activity Completed		

Lesson 13.10 Margin of Error Assignment Comments: For #1, express your answers to the nearest thousandth. For #2a, #2b, and #2c, express your answers to the nearest hundredth. For #2d, express your answers to the nearest whole number For #4a, express your answers for the σ_p to the nearest thousandth and express your answers to the m.o.e to the nearest tenth of a percent.					
	Video Watched		Lesson Completed		Assignment Completed

Practice with Margin of Error and 95% Confidence Intervals Activity

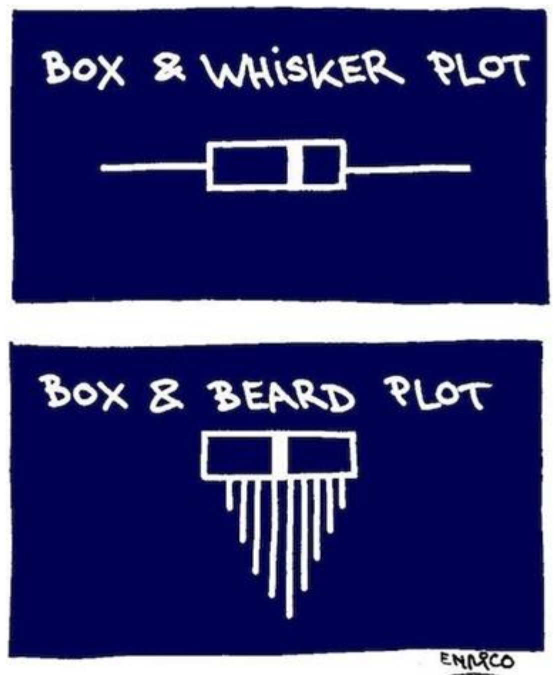
Activity Comments:
For #1, express your answers to the nearest thousandth.
For #5, express answer to the nearest percent.
For 7, express answer to the nearest tenth of a percent.

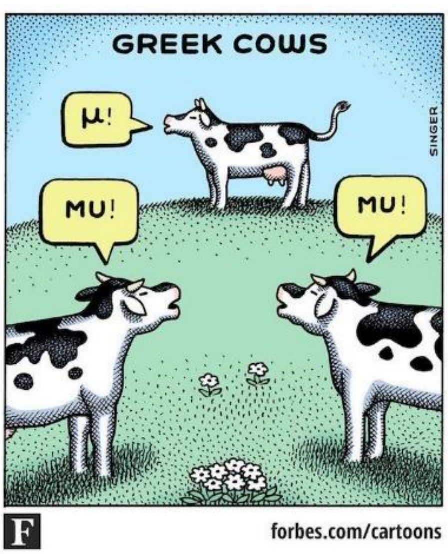
			Activity Completed		

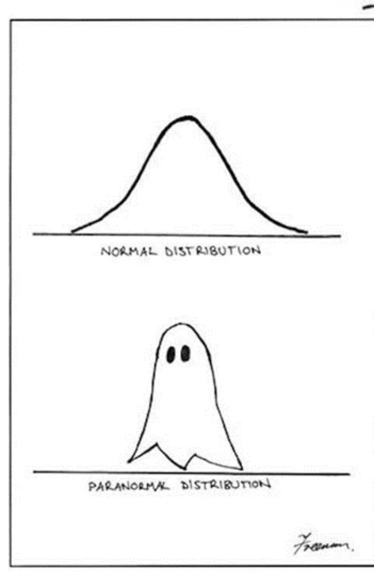
Unit 13 Test

Test Comments:
None

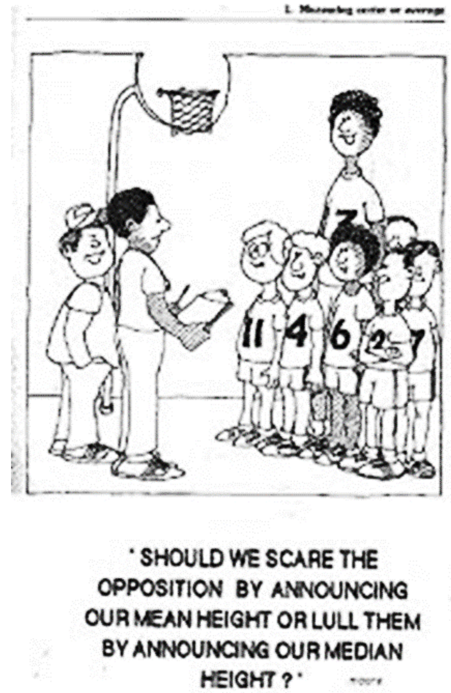
			Test Completed		

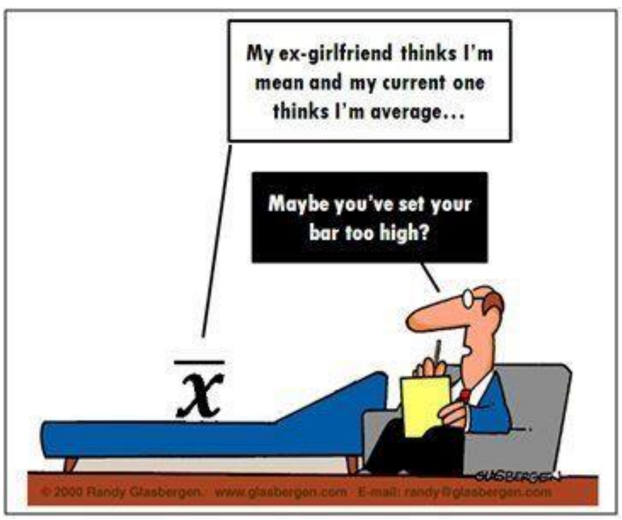







DEPARTMENT OF PROBABILITY
VERY LIKELY TO BE BACK IN AN HOUR
CARTOONSTOCK .com
Search ID: msen295
Seddon..


' SHOULD WE SCARE THE OPPOSITION BY ANNOUNCING OUR MEAN HEIGHT OR LULL THEM BY ANNOUNCING OUR MEDIAN HEIGHT ? '


My ex-girlfriend thinks I'm mean and my current one thinks I'm average...
Maybe you've set your bar too high?
x̄
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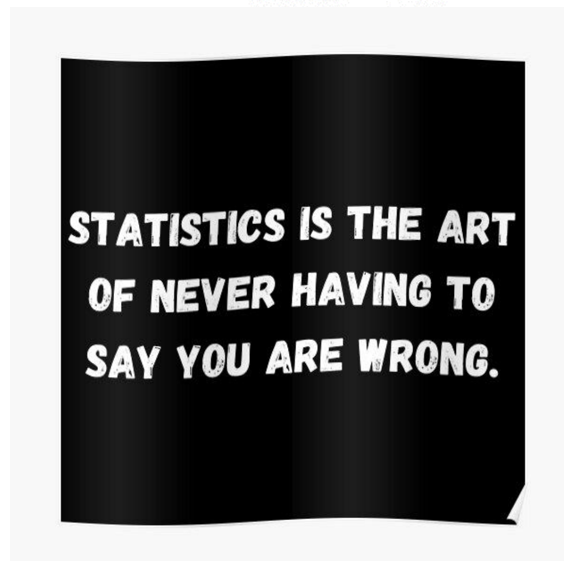
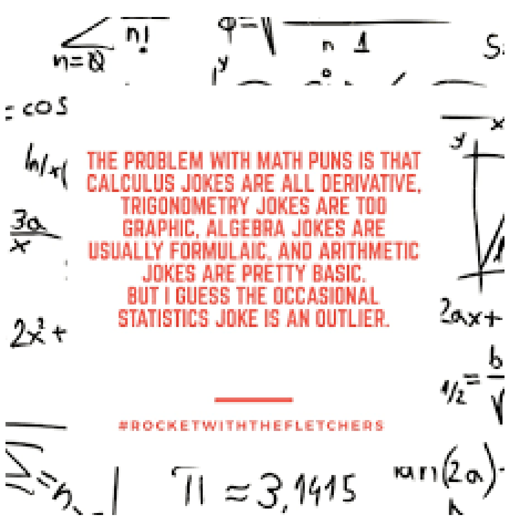
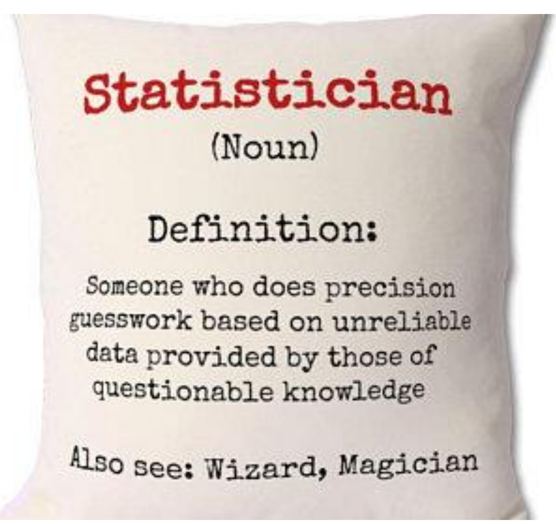

STATISTICS IS THE ART OF NEVER HAVING TO SAY YOU ARE WRONG.


THE PROBLEM WITH MATH PUNS IS THAT CALCULUS JOKES ARE ALL DERIVATIVE, TRIGONOMETRY JOKES ARE TOO GRAPHIC, ALGEBRA JOKES ARE USUALLY FORMULAIC, AND ARITHMETIC JOKES ARE PRETTY BASIC.
BUT I GUESS THE OCCASIONAL STATISTICS JOKE IS AN OUTLIER.
#ROCKETWITHTHEFLETCHERS


Statistician
(Noun)
Definition:
Someone who does precision guesswork based on unreliable data provided by those of questionable knowledge
Also see: Wizard, Magician

Name: ___________________________________ Date: __________________

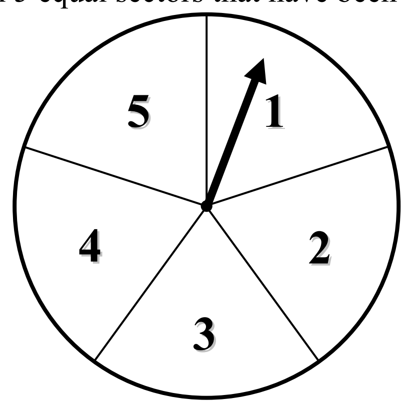
INTRODUCTION TO PROBABILITY
COMMON CORE ALGEBRA II

Mathematics seeks to quantify and model just about everything. One of the greatest challenges is to try to quantify chance. But that is exactly what probability seeks to do. With probability, we attempt to assign a number to how likely an **event** is to occur. Terminology in probability is important, so we introduce some basic terms here:

BASIC PROBABILITY TERMINOLOGY

1. **Experiment:** Some process that occurs with well defined outcomes.
2. **Outcome:** A result from a single **trial** of the experiment.
3. **Event:** A collection of one or more outcomes.
4. **Sample Space:** A collection of all of the outcomes of an experiment.

***Exercise* #1:** An experiment is run whereby a spinner is spun around a circle with 5 equal sectors that have been marked off as shown.



(a) What is the **experiment**?

(b) Give one outcome of the experiment.

(c) What is the probability of spinning the spinner and landing on an odd number? What is the event here? What outcomes fall into the event?

The answer from (c) helps us to define the basic formula that dictates all probability calculations:

THE BASIC DEFINITION OF PROBABILITY

The probability of an event E occurring is given by the ratio: $P(E)=\frac{n(E)}{n(S)}$, where:

$n(E)$ is the number of outcomes that fall into the event E

$n(S)$ is the number of outcomes that fall into the sample space

***Exercise* #2:** Given the above definition, between what two numbers must ALL probabilities lie? Explain.

When we deal with **theoretical probability** we don't actually have to run the experiment to determine the probability of an event. We simply have to know the number of outcomes in the sample space and the number of outcomes that fall into our event. Let's take a look at a slightly more challenging scenario.

***Exercise* #3:** A fair coin is flipped three times and the result is noted each time. The sample space consists of **ordered triples** such as (H, H, T), which would represent a head on the first toss, a head on the second toss, and a tail on the third toss.

(a) Draw a **tree diagram** to show all of the different outcomes in the sample space.

(b) List all of the outcomes as ordered triples. How many of them are there?

(c) Find each of the following probabilities based on your answers from (a) and (b):

(i) $P(\text{all heads})$

(ii) $P(\text{exactly 2 heads})$

(iii) $P(\text{all heads or all tails})$

Sometimes we have to quantify chance by using observations that have been made in the real-world. In this case we talk about **empirical probability**. The fundamental equation for probability still stands.

***Exercise* #4:** A survey was done by a marketing company to determine which of three sodas was preferred by people in a blind taste test. The results are shown below.

Soda	Number who Preferred
A	18
B	24
C	11
Total	53

(a) Find the empirical probability that a person selected at random from this group would prefer soda B. Express your answer as a fraction and as a decimal accurate to two decimal places (the standard).

(b) Find the empirical probability that a person selected at random from this group would *not* prefer soda A. Again, express your answer as a fraction and as a decimal accurate to two decimal places.

Name: ____________________________________ Date: __________________

Introduction to Probability
Common Core Algebra II Homework

Fluency

1. Which of the following could *not* be the value of a probability? Explain your choice.

   (1) 53% (3) $\frac{5}{4}$

   (2) 0.78 (4) $\frac{3}{4}$

   ____

2. If a month is picked at random, which of the following represent the probability its name will begin with the letter J?

   (1) 0.08 (3) 0.12

   (2) 0.25 (4) 0.33

   ____

3. If a coin is tossed twice, which of the following gives the probability that it will land both times heads up or both times tails up?

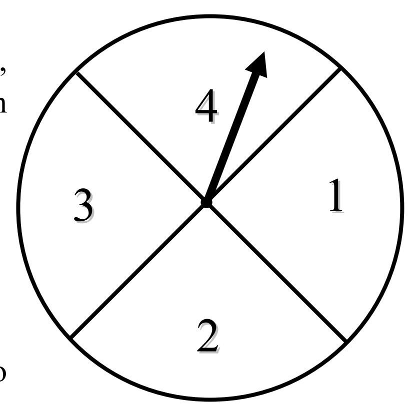
   (1) 0.75 (3) 0.25

   (2) 0.67 (4) 0.50

   ____

4. A spinner is now created with four equal sized sectors as shown. An experiment is run where the spinner is spun twice and the outcome is recorded each time.

   (a) Create a sample space list of ordered pairs that represent the outcomes, such as $(4,2)$, which represent spinning a 4 on the first spin and a 2 on the second spin.



   (b) Using your answer from (a), determine the probability of obtaining two numbers with a sum of 4.

APPLICATIONS

5. Samuel pulls two coins out of his pocket randomly without replacement. If his pocket contains one nickel, one dime, and one quarter, what is the probability that he pulled more than 20 cents out of his pocket? Justify your work by creating a tree diagram or a sample space.

6. Janice, Tom, John, and Tamara are trying to decide on who will make dinner and who will wash the dishes afterwards. They randomly pull two names out of a hat to decide, where the first name drawn will make dinner and the second will do the dishes. Determine the probability that the two people pulled will have first names beginning with the same letter. Assume the same person cannot be picked for both.

7. A blood collection agency tests 50 blood samples to see what type they are. Their results are shown in the table below.

(a) If a blood sample is picked at random, what is the probability it will be type B?

(b) If a blood sample is picked at random, what is the probability it will not be type O?

Blood Type	Number of Samples
O	18
A	22
B	7
AB	3
Total	50

(c) Are the two probabilities you calculated in (a) and (b) **theoretical** or **empirical**? Explain your choice.

Name: ___________________________________ Date: _________________

SETS AND PROBABILITY
COMMON CORE ALGEBRA II

Since the basic calculations within probability involve counting the number of **outcomes** that fit into particular **events**, it makes sense to have a tool to visualize and keep track of all of the outcomes in a **sample space**. We will do this by using sets. Recall the basic definition of a set:

SET DEFINITION

A **set** is simply a collection of things (numbers, objects, etcetera) that satisfy a well-defined criteria. The things that are contained in the set are called the **elements** of the set

***Exercise* #1:** The set A is defined as the collection of all integers that are greater than 0 and less than 10.

(a) Write out set A in **roster form**.

(b) Show set A in **Venn Diagram** form. This will be a very simple Venn Diagram.

(c) A **subset** is any set whose elements are all contained within another set. Give two possible rules that could define subsets of A and then write the sets as B and C in roster form. Do sets B and C have any elements in common?

Set B's Definition: ___________________________________ $B=$

Set C's Definition: __________________________________ $C=$

Let's get back to a bit of probability.

***Exercise* #2:** Consider an experiment where we first toss a coin and note the outcome and then roll a six-sided die and note the outcome.

(a) Write a set of ordered pairs, such as $(H,4)$, that represents all outcomes for this experiment. Recall that this is called the **sample space**. We will generally call this set S.

(b) Write a set of ordered pairs that represents the event of getting a tail and an even number. Call this set A.

(c) The complement of a set A will be all of the events in the sample space S that do not fall into set A. Write out the complement of set A. We'll call this set B.

(d) Find $P(A)$ and $P(B)$.

A set and its complement are important in probability because all outcomes either fall into an event or into its complement, but not both (called **mutually exclusive**). Different textbooks use different notations to denote complements. Since the notation is not universal, we will simply refer to complements by name instead of by symbol.

***Exercise* #3:** Consider rolling a single six-sided die and recording the result. Let set A be the event of rolling a number greater than 4 and let set B be the complement of set A.

(a) Draw a Venn Diagram that illustrates the sample space, S, and sets A and B.

(b) Find $P(A)$ and $P(B)$.

(c) What is true of the sum $P(A)+P(B)$?

(d) Prove that the sum of the probability of an event with the probability of its complement will always be 1.

We use the relationship developed in (d) all the time without even thinking about it. Try the following.

***Exercise* #4:** Answer each of the following problems by using the relationship developed in Exercise #3(d).

(a) If the probability I will draw a red marble from a bag is $\frac{3}{17}$, what is the probability that I won't draw a red marble from a bag?

(b) If the probability that it will rain tomorrow is 20%, what is the probability that it won't rain tomorrow?

In theoretical probability calculations, the sets that make up the sample spaces can get difficult to write out. It is good to remember things like tree diagrams to help.

***Exercise* #5:** Two four-sided die are rolled and the number on each is noted.

(a) Draw a tree diagram that represents all outcomes in the sample space. How many are there?

(b) What is the probability that you don't get two of the same number?

Name: ____________________________________ Date: __________________

SETS AND PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

APPLICATION

1. Consider the experiment of picking one of the 12 months at random.

   (a) Write down that sample space, S, for this experiment. What is the value of $n(S)$?

   (b) Let E be the event (set) of picking a month that begins with the letter J. Write out the elements of E.

   (c) What is the probability of E, i.e. $P(E)$?

   (d) What is the probability of picking a month that does *not* start with the letter J?

2. Consider the set, A, of all integers from 1 to 10 inclusive (that means the 1 and the 10 are included in this set). Give a set B that is a subset of A. State its definition and list its elements in roster form. Then give a set C that is the complement of B.

   Set B's Definition: ____ Set B:

   Set C:

3. If A and B are complements, then which of the following is true about the probability of B based on the probability of A?

   (1) $P(B)=P(A)+1$

   (3) $P(B)=\dfrac{1}{P(A)}$

   (2) $P(B)=1-P(A)$

   (4) $P(B)=P(A)-1$

   ____

4. If a fair coin is flipped three times, the probability it will land heads up all three times is $\frac{1}{8}$. Which of the following is the probability that when a coin is flipped three times at least one tail will show up?

   (1) $\frac{7}{8}$

   (3) $\frac{3}{2}$

   (2) $\frac{1}{8}$

   (4) $\frac{1}{2}$

   ____

5. A four-sided die, in the shape of a tetrahedron, is rolled twice and the number rolled is recorded each time.

(a) Draw a tree-diagram that shows the sample space, S, of this experiment. How many elements are in S?

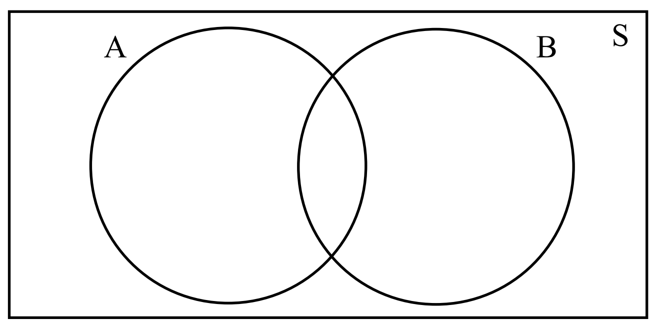
(b) Let E be the event of rolling two numbers that have an odd product. List all of the elements of E as ordered pairs.

(c) What is the probability that the two rolled numbers have a product that is odd?

(d) What is the probability that the two rolled numbers have a product that is even?

REASONING

6. Consider the set of all integers from 1 to 10, i.e. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, to be our sample space, S. Let A be the set of all integers in S that are even and let B be the set of all integers in S that are multiples of 3. Fill in the circles of the Venn diagram with elements from S. If an element lies in both sets, place it in the overlapping region.



7. Find in the following:

$n(A)=$ $\quad\quad$ $n(B)=$

8. Why is the following equation *not* true? Explain.

$$n(S)=n(A)+n(B)$$

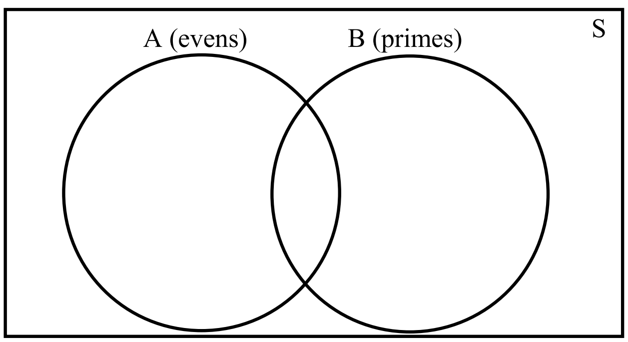
Name: ____________________________________ Date: __________________

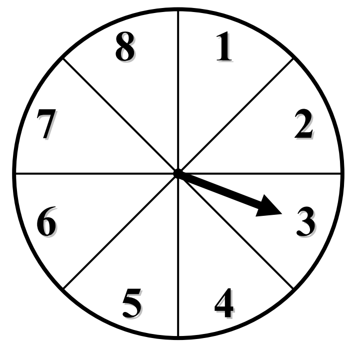
ADDING PROBABILITIES
COMMON CORE ALGEBRA II

There are times that we want to determine the probability that either event A happened or event B happened. To do this, we need to be able to account for all of the outcomes that fall into either one of the two events. Let's see how this looks given a simple Venn diagram.

***Exercise* #1:** Consider the spinner shown below that has been divided into eight equally sized sectors of a circle. The spinner is spun once. In this experiment we will let A be the event of it landing on an even and B be the event of it landing on a prime number.

Fill in the Venn Diagram below with the actual numbers from the spinner.





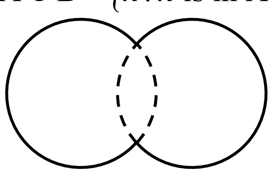
When we have two (or more) sets, we can talk about their **union** and their **intersection**. Their technical definitions are given below.

THE UNION AND INTERSECTION OF TWO SETS

For two sets, A and B, their **union**, OR, and their **intersection**, AND, are given by:

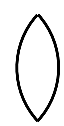
(1) **Union**:

$\text{A or B} = \text{A} \cup \text{B} = \{x : x \text{ is in A or } x \text{ is in B}\}$



(2) **Intersection:**

$\text{A and B} = \text{A} \cap \text{B} = \{x : x \text{ is in A and } x \text{ is in B}\}$



***Exercise* #2:** From Exercise #1 write out the following two sets:

(a) A or B (The Union):

(b) A and B (The Intersection):

***Exercise* #3:** From Exercise #2, why is the equation $n(\text{A or B}) = n(\text{A}) + n(\text{B})$ generally *not* true? What would be the correct modification to make it true? Use the last example to help explain.

Two-way frequency charts give us a great example of how **events or sets can combine (union) and overlap (intersection).** Let's take a look at this and develop some ideas about probability along the way.

***Exercise* #4:** A small high school surveyed 52 of its seniors about their plans after they graduate. They found the following data and wanted to analyze it based on gender. In this case, if we pick a student at random we can place them into one of four events:

M = Male F = Female

C = Going to College N = Not going to college

	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

(a) Give the values for each of the following:

(i) $n(\text{M})=$ (ii) $n(\text{F})=$ (iii) $n(\text{C})=$ (iv) $n(\text{N})=$

(v) $n(\text{M and C})=$ (vi) $n(\text{F and C})=$ (vii) $n(\text{F or C})=$

(b) What is the probability that a person picked at random would be a female who is going to college? Represent this using either a union or an intersection.

(c) What is the probability that a person picked at random would be a female or someone going to college? Represent this using either a union or an intersection.

(d) Explain why $P(\text{F or C}) \neq P(\text{F}) + P(\text{C})$?

(e) Fill in the general probability law based on (d):

$P(\text{A or B})=$

Sometimes we can avoid the probability law that we encounter in (e) by simply keeping careful track of what elements of the sample space are in both of our sets and making sure we don't count any element twice.

***Exercise* #5:** A standard six-sided die is rolled once. Find the probability that the number rolled was either an even or a multiple of three. Represent this problem and the sets involved using a Venn diagram. Even though you don't need it, verify the **probability addition rule** from Exercise #4 (e).

There are some situations, though, where the **probability addition rule** is unavoidable.

***Exercise* #6:** Insurance companies typically try to sell many different policies to the same customers. At one such company, 56% of all of the customers have car insurance policies, 48% have home insurance policies, and 18% have both. A customer is picked at random.

(a) Find the probability that she or he has at least one of the policies.

(b) Find the probability that she or he has neither of the policies.

Name: ____________________________________ Date: __________________

ADDING PROBABILITIES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given the two sets below, give the sets that represent their union and their intersection.

$$A = \{3, 5, 7, 9, 11, 13\} \qquad B = \{1, 5, 9, 13, 17\}$$

(a) Union: A or B =

(b) Intersection: A and B =

2. Using sets A and B from #1, verify the addition law for the union of two sets:

$$n(\text{A or B}) = n(\text{A}) + n(\text{B}) - n(\text{A and B})$$

APPLICATIONS

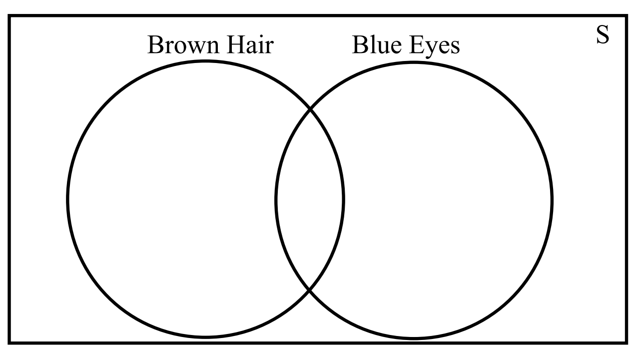
3. Red Hook High School has 480 freshmen. Of those freshmen, 333 take Algebra, 306 take Biology, and 188 take both Algebra and Biology. Which of the following represents the number of freshmen who take at least one of these two classes?

(1) 639 (3) 451

(2) 384 (4) 425

____

4. Evie was doing a science fair project by surveying her biology class. She found that of the 30 students in the class, 15 had brown hair and 17 had blue eyes and 6 had neither brown hair nor blue eyes. Determine the number of students who had brown hair and blue eyes. Use the Venn Diagram below to help sort the students if needed.



5. A standard six-sided die is rolled and its outcome noted. Which of the following is the probability that the outcome was less than three or even?

(1) $\frac{2}{3}$ (3) $\frac{5}{6}$

(2) $\frac{1}{3}$ (4) $\frac{1}{6}$ __________

6. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then which of the following represents the probability that a day in early March would have either rain or snow?

(1) 0.30 (3) 0.02

(2) 0.34 (4) 0.26 __________

7. A survey was done of students in a high school to see if there was a connection between a student's hair color and her or his eye color. If a student is chosen at random, find the probability of each of the following events.

(a) The student had black hair.

(b) The student had blue eyes.

(c) The student had brown eyes and black hair.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(d) The student had blue eyes or blond hair.

(e) The student had black hair or blue eyes.

8. A recent survey of the Arlington High School 11th grade students found that 56% were female and 58% liked math as their favorite subject (of course). If 76% of all students are either female or liked math as their favorite subject, then what percent of the 11th graders were female students who liked math as their favorite subject? Show how you arrived at your answer.

Name: ______________________________________ Date: ___________________

CONDITIONAL PROBABILITY
COMMON CORE ALGEBRA II

When the probability of one event occurring changes depending on other events occurring then we say that there is a **conditional probability**. The language and symbolism of conditional probability can be a bit confusing, but the idea is fairly straightforward and can be developed with two-way frequency charts.

***Exercise* #1:** Let's revisit a two-way frequency chart we saw in the last lesson. In this study, 52 graduating seniors were surveyed as to their post-graduation plans and then the results were sorted by gender.

Let the following letters stand for the following events.

M = Male F = Female

C = Going to College N = Not going to college

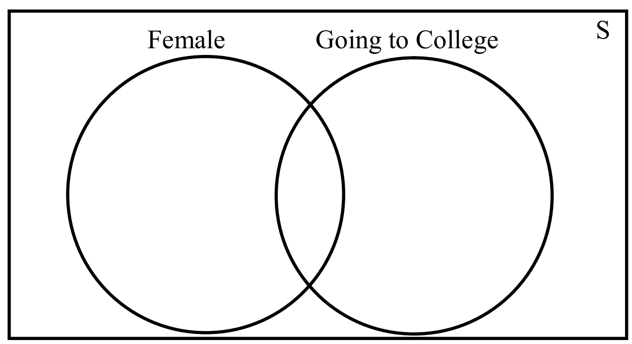
	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

If a person was picked at random, find the probability that the person was

(a) a female, i.e. $P(\text{F})$

(b) going to college $P(\text{C})$

(c) going to college **given** they are female, i.e. $P(\text{C} \mid \text{F})$. Draw a Venn diagram below to help justify the ratio that you give as the probability.



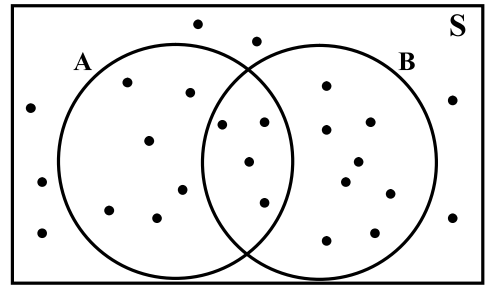
(d) Which is more likely, that a person picked at random will be going to college, given they are a male, i.e. $P(\text{C} \mid \text{M})$, or that a person will be male, given they are going to college, i.e. $P(\text{M} \mid \text{C})$. Show that calculations for both.

$P(\text{C} \mid \text{M})$

$P(\text{M} \mid \text{C})$

We can generalize the process of calculating **conditional probabilities** based on counts and a way to calculate these probabilities based on other probabilities.

***Exercise* #2:** In the generic Venn diagram shown to the right each dot represents an equally likely outcome of the sample space. Some of these fall only into event A, some only into event B, some in both events and some in neither.



(a) Consider the probability of A occurring given that B has occurred. Give a formula for this probability based on counting the number of elements in each set and their intersection.

$P(\text{A} \mid \text{B}) =$

(b) Divide both of the numerator and denominator in (a) by the number of total elements in the sample space. Then rewrite the formula in (a) in terms of probabilities instead of counts.

$P(\text{A} \mid \text{B}) =$

It's great when we can count elements that lie in events and their intersection, but sometimes we cannot. For example, let's revisit a relative frequency table that we saw in a previous homework.

***Exercise* #3:** A survey was taken to examine the relationship between hair color and eye color. The chart below shows the proportion of the people surveyed who fell into each category. If a person was picked at random, find each of the following conditional probabilities. Show the calculation you used.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) Find the probability the person picked had brown eyes given they had blond hair.

$P(\text{brown eyes} \mid \text{blond hair})$

(b) Find the probability the person had red hair given they had green eyes.

$P(\text{red hair} \mid \text{green eyes})$

(c) Does having red hair seem have some **dependence** on having green eyes? How can you tell or quantify this dependence?

Name: ______________________________________ Date: ___________________

CONDITIONAL PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given that $P(\text{B} \mid \text{A})$ means the probability of event B occurring given that event A will occur or has occurred, which of the following correctly calculates this probability?

(1) $\dfrac{P(\text{B})}{P(\text{A})}$ (3) $\dfrac{P(\text{A})}{P(\text{B})}$

(2) $\dfrac{P(\text{A and B})}{P(\text{B})}$ (4) $\dfrac{P(\text{A and B})}{P(\text{A})}$

____

APPLICATIONS

2. Of the 650 juniors at Arlington High School, 468 are enrolled in Algebra II, 292 are enrolled in Physics, and 180 are taking both courses at the same time. If one of the 650 juniors was picked at random, what is the probability they are taking Physics if we know they are in Algebra II?

(1) 0.38 (3) 0.45

(2) 0.62 (4) 0.58

____

3. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then calculate each of the following:

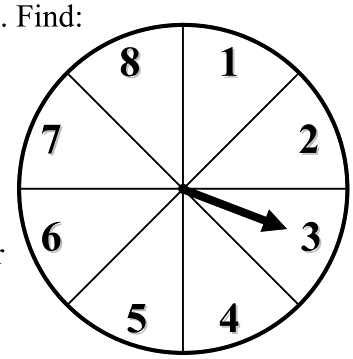
(a) the probability it will rain given that it is snowing, i.e.

$P(\text{rain} \mid \text{snow})$

(b) the probability it will snow given that it is raining, i.e.

$P(\text{snow} \mid \text{rain})$

4. A spinner is spun around a circle that is divided up into eight equally sized sectors. Find:



(a) $P(\text{perfect square} \mid \text{even})$ (b) $P(\text{odd} \mid \text{prime})$

(c) What is more likely: getting a multiple of four given we spun an even or getting an odd, given we spun a number greater than 2? Support your answer.

5. A survey was done of commuters in three major cities about how they primarily got to work. The results are shown in the frequency table below. Answer the following conditional probability questions.

	Car	Train	Walk	Total
New York	.05	.25	.10	.40
Los Angeles	.18	.12	.05	.35
Chicago	.08	.14	.03	.25
Total	.31	.51	.18	1.00

(a) What is the probability that a person picked at random would take a train to work given that they live in Los Angeles.

$P(\text{train} \mid \text{LA})$

(b) What is the probability that a person picked at random would live in New York given that they drive a car to work.

$P(\text{NYC} \mid \text{Car})$

(c) Is it more likely that a person who takes a train to work lives in Chicago or more likely that a person who lives in Chicago will take a train to work. Support your work using conditional probabilities.

REASONING

6. The formula for conditional probability is: $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$. Solve this formula for $P(\text{A and B})$.

7. We say **two events**, A and B, **are independent** if the following is true:

$$P(\text{B} \mid \text{A}) = P(\text{B}) \text{ and likewise } P(\text{A} \mid \text{B}) = P(\text{A})$$

Interpret what the definition of **independent events** means in your own words.

Name: ___________________________________

UNIT #12 – CONDITIONAL PROBABILITY - ADDITIONAL PRACTICE
COMMON CORE ALGEBRA II

1. On any given day, the probability that Jeremiah goes for a run and has homework to complete is 0.55. If there is a 0.85 probability that he goes running and a 0.65 probability that he has homework, determine the probability that Jeremiah goes running given that he has homework. Give your answer to the nearest tenth.

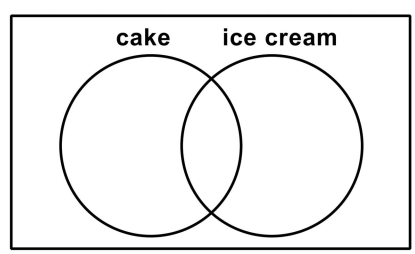
2. At a local restaurant, 52% of the employees work both nights and weekends. If 63% of the employees work nights, what percent, to the nearest tenth, of the employees who work nights are working weekends?

3. In a group of 190 students, 112 students are taking a tech course, 74 are taking an art class, and 62 are taking both courses. If one student is randomly chosen from the group, what is the probability that they are taking tech given that they are taking art? Express your answer to the nearest tenth of a percent.

4. Use the data in the table to determine the if it is more likely that someone drinks coffee given that they stay up late or more likely that someone stays up late given that they drink coffee. Justify your answer.

	Sleep early	Up late
Coffee	33	46
No coffee	25	42

5. At a birthday party, 11 guests wanted cake and ice cream, 9 asked for only a piece of cake, 5 just wanted ice cream, and 2 of the guests did not want anything at all.



(a) Complete the Venn Diagram using the information given above.

(b) What is the probability that a randomly selected guest wants ice cream given that they did not want cake?

6. Several adults and children were observed to see how many of them were wearing glasses. The results are summarized in the relative frequency table. If a person from this group is randomly selected, answer each of the following questions.

	Glasses	No glasses	Total
Adult	0.20	0.38	0.58
Child	0.17	0.25	0.42
Total	0.37	0.63	1.00

(a) To the nearest hundredth, determine the probability that a person who wears glasses is an adult.

(b) What is the probability, to the nearest hundredth, that a child picked at random does not wear glasses?

7. A survey was given to a group of students to see whether they prefer scary movies or rollercoaster rides. The results were broken down by grade and are summarized in the table below.

	Scary movies	Rollercoasters
9^{th} grade	22	30
10^{th} grade	19	14
11^{th} grade	31	29
12^{th} grade	25	42

(a) What is the probability that a randomly selected student prefers rollercoasters given that they are not in 9^{th} grade?

(b) How many of the students surveyed are in 11^{th} grade and love rollercoasters?

(c) What is the probability of randomly selecting a 10^{th} grader given that they like scary movies?

(d) If a student is randomly selected, what is the probability that they are in 9th grade or love scary movies?

(e) If the randomly selected student is in 12^{th} grade, what is the probability that they prefer rollercoasters?

Name: ___________________________________

Score: ________ / **22**

UNIT #12 QUIZ (THROUGH LESSON #4)
COMMON CORE ALGEBRA II

PART I QUESTIONS: Answer all questions in this part by writing the choice of the appropriate answer in the blank beside the problem. Each question is worth 2 points. No partial credit will be awarded.

1. Jillian is going to randomly choose one book from a shelf of books that contains five mysteries, two autobiographies and four science fiction novels. Which of the following is closest to the probability that Jillian does not choose an autobiography?

(1) 0.18　　(3) 0.82

(2) 0.67　　(4) 0.91　　________

2. A standard fair die is rolled one time. What is the probability that the number it lands on is less than 4 or even?

(1) $\frac{2}{6}$　　(3) $\frac{4}{6}$

(2) $\frac{3}{6}$　　(4) $\frac{5}{6}$　　________

3. If A and B are complements, $P(\text{A}) = a$, and $P(\text{B}) = b$, which of the following must be true?

(1) $a = b + 1$　　(3) $a = 1 - b$

(2) $a = \frac{1}{b}$　　(4) $a = b - 1$　　________

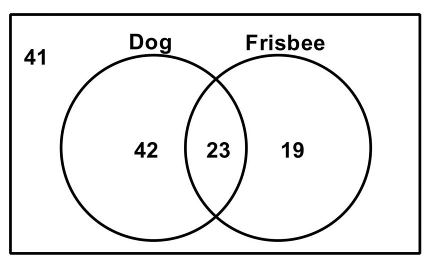
4. In a survey, 125 people were asked if they have a dog and if they have played frisbee at least once in the past month. Using the results shown in the Venn Diagram below, which of the following is closest to the probability that a person played frisbee given that they do not have a dog?

(1) 0.297

(2) 0.317

(3) 0.463

(4) 0.506



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5. If a fair coin is flipped four times, the probability that it lands tails up all four times is $\frac{1}{16}$. Which of the following is the probability that when a coin is flipped four times, it will land on heads at least once?

(1) $\frac{1}{16}$　　(3) $\frac{8}{16}$

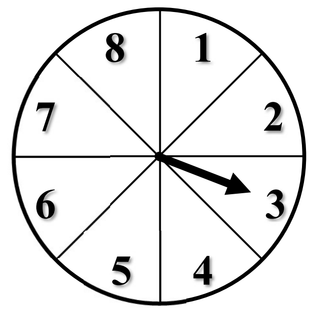
(2) $\frac{4}{16}$　　(4) $\frac{15}{16}$　　________

PART II QUESTIONS: Answer all questions in this part. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

6. A survey asked participants if they liked cheese pizza, pepperoni pizza, both kinds, or neither kind. Of those surveyed, 65% liked cheese, 72% liked pepperoni, and 54% liked both. If 200 people were surveyed, how many people liked either cheese or pepperoni pizza? [4 points]

7. A fair spinner is made up of 8 sectors of the same size. If the spinner is spun once, determine the probability of each of the following events. [4 points]

(a) The spinner landed on a prime number.



(b) The spinner landed on a 7 and a number less than 4.

(c) The spinner landed on an even or a 5.

(d) The spinner landed on a 4 or a multiple of 2.

8. A group of people were asked if they have any siblings and if they have any pets. Using the results of the survey that are summarized in the table below, determine the answer to each of the following questions:

(a) What is the probability that a randomly selected person from this group has a pet and at least one sibling? [1 point]

	Pet	No pet
Sibling(s)	52	37
No Siblings	43	41

(b) Which is more likely: a person who has a pet also has at least one sibling or a person who does not have any siblings does not have a pet? Support your answer. [3 points]

Name: ____________________________________ Date: __________________

INDEPENDENT EVENTS
COMMON CORE ALGEBRA II

In the previous lesson's homework we saw how the occurrence of one event could change the probability of another event. When this happens, we say the two events are **not independent** of one another. When the occurrence of one event has no effect on the probability of another event happening, we say the events are **independent**.

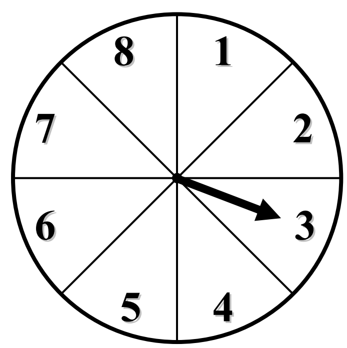
***Exercise* #1:** Classify each of the following scenarios as having events that are dependent or events that are independent.

(a) A person pulls a red marble out of a bag that has 5 blue and 7 red marbles and does not replace it. Then a person pulls another red marble. Is the probability of pulling the second red marble out dependent on pulling the first red marble? Explain.

(b) A person flips a coin and notes that it comes up heads. Then the person rolls a standard six-sided die and notes that it comes up as a number less than three. Is the probability that the number came up less than three dependent on getting a head when flipping the coin? Explain.

The idea of **independence** is one that comes fairly naturally, but is important in order to see if there are associations amongst two events. Let's develop a tool to test dependence.

***Exercise* #2:** The spinner below is spun once and its outcome is noted. Let E be the event of getting an even, let P be the event of getting a prime, and let L be the event of getting a number less than 5. Find the following probabilities:



(a) The probability of getting an even, i.e. $P(\mathrm{E})$.

(b) The probability of getting an even given that the outcome was a prime number, i.e. $P(\mathrm{E}\mid\mathrm{P})$.

(c) The probability of getting an even given that the outcome was a number less than 5, i.e. $P(\mathrm{E}\mid\mathrm{L})$.

(d) Which event does E depend on, P or L? How can you tell? What is a reasonable test?

DEFINITION OF INDEPENDENT EVENTS

Two events, A and B, are defined to be independent if:

$$P(\text{A} \mid \text{B}) = P(\text{A}) \quad \text{and likewise} \quad P(\text{B} \mid \text{A}) = P(\text{B})$$

***Exercise* #3:** A survey of 57 sixth graders was done to determine which subject was their favorite. The results are shown in the table below sorted by gender.

	Math	English	Social Studies	Science	Total
Female	8	6	10	6	30
Male	10	4	9	4	27
Total	18	10	19	10	57

(a) Does it appear, based on the data in this table, that the preference for math as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

(b) Does it appear, based on the data in this table, that the preference for social studies as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

There is a nice **test** for **independence** that can be applied easily and comes from our formula for conditional probability from the last lesson.

***Exercise* #4:** Given that $P(\text{A} \mid \text{B}) = \dfrac{P(\text{A and B})}{P(\text{B})}$, do the following.

(a) If A and B are independent, then rewrite this formula and solve for $P(\text{A and B})$.

(b) The probability that a person is left handed is 12%, the probability they have brown eyes is 42%, and the probability they have brown eyes and are left handed is 2%. Is the event of having brown eyes independent of being left handed? Support your answer.

THE PRODUCT TEST FOR INDEPENDENCE

Name: ___________________________________ Date: __________________

INDEPENDENT EVENTS
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

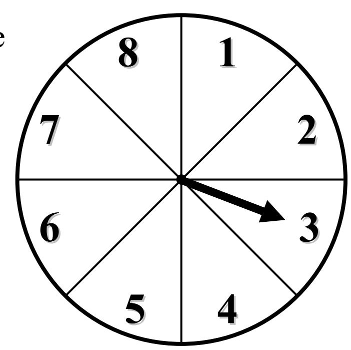
1. In each of the following, a scenario is given with two events. Explain whether these events are independent or dependent.

(a) A coin is flipped and lands on a head. The coin is flipped a second time and lands on its head again. Is the probability of it landing on heads the second time dependent on it landing on head the first time? Explain.

(b) An elementary class consists of 8 boys and 10 girls. A child is chosen at random and it is a girl. A second child is randomly chosen again from the remaining children and it is a boy. Was the probability of choosing the boy dependent on choosing a girl first? Explain.

2. A newspaper did a survey of adults and found that 54% of the population as a whole favored stricter gun control laws. They broke down the results along gender lines and found that 65% of women favored stricter laws while only 44% of men favored them. If a person was selected at random, are the events of being a woman and being in favor of stricter gun control laws dependent or independent? Explain.

3. The eight-sector spinner is back. If the spinner is spun once and the outcome is noted answer the following questions.



(a) Let the event S be the event of getting a perfect square, i.e. 1 or 4. What is the probability of getting a perfect square, i.e. $P(\text{S})$?

(b) Let E be the event of getting an even. What is the probability of getting a perfect square given you got an even, i.e. $P(\text{S} \mid \text{E})$? Are the two events independent? Explain.

(c) Let M be the event of getting a multiple of four. What is the probability of getting a perfect square given that you got a multiple of four, i.e. $P(\text{S} \mid \text{M})$? Are the two events independent? Explain.

4. If two events, A and B, and independent then $P(\text{A and B}) =$

(1) $\dfrac{P(\text{A})}{P(\text{B})}$ (3) $\dfrac{P(\text{B})}{P(\text{A})}$

(2) $P(\text{A})\cdot P(\text{B})$ (4) $P(\text{A})+P(\text{B})$ __________

5. There is a 34% chance that a person picked at random from the adult population is regular smoker of cigarettes and an 18% chance that a person picked has emphysema. If the percent of the adult population that are both regular smokers and suffer from emphysema is 14%, is being a smoker independent from having emphysema? Justify your result by using the **Product Test for Independence**.

6. The two-way frequency table below shows the proportions of a population that have given hair color and eye color combinations. Use this table to answer the following.

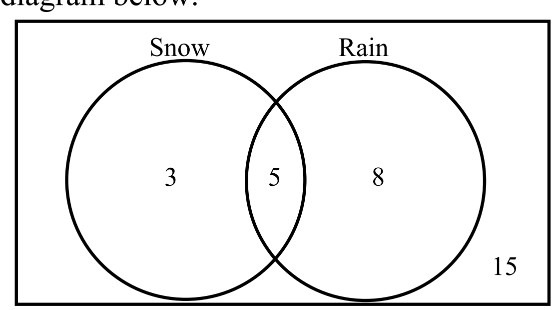
(a) Show that the events of having green eyes and red hair are dependent.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.17	0.21	0.02	0.40
	Brown	0.21	0.13	0.01	0.35
	Green	0.07	0.03	0.15	0.25
	Total	0.45	0.37	0.18	1.00

(b) Many of the hair colors have dependence on eye color. Does having blond hair have a dependence on having brown eyes? Show the analysis that leads to your decision.

7. The month of March has 31 days in it. In New York, March has days when it snows, days when it rains, and days when it does both. This breakdown is shown in the Venn diagram below.

Based on the diagram, are the events of having snow and having rain dependent or independent? Justify.



Name: ___________________________________ Date: _________________

MORE WORK WITH INDEPENDENCE – ADDITIONAL PRACTICE
COMMON CORE ALGEBRA II

1. Given events J and K, such that $P(\text{J}) = 0.9$, $P(\text{K}) = 0.5$, and $P(\text{J and K}) = 0.3$, determine whether J and K are independent or dependent events. Justify your answer.

2. A random integer is chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Explain why choosing a prime number is not independent of choosing an odd number.

3. The probability that someone likes ice cream is 0.5 and the probability that they like chocolate is 0.7. If the probability that someone likes ice cream or chocolate is 0.85, determine whether the events of liking ice cream and liking chocolate are independent or dependent.

4. A standard six-sided die is rolled once, and its outcome is noted. Let E be the event of rolling an even number. Let G be the event of rolling a number greater than 2. Are the two events independent? Explain.

5. If two events, A and B, are independent, which of the following statements is *not* always true?

   (1) $P(\text{A}) = P(\text{A}|\text{B})$

   (2) $P(\text{A or B}) = P(\text{A}) + P(\text{B})$

   (3) $P(\text{B}|\text{A}) = P(\text{B})$

   (4) $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$

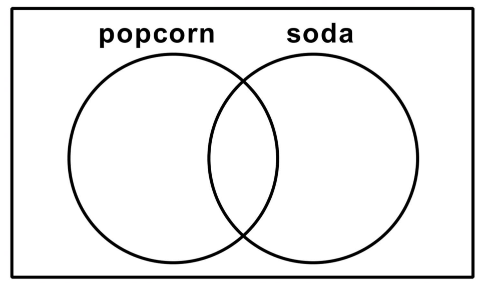
____

6. A group of people were observed at a school football game to see how many people were wearing spirit gear. The two-way frequency table below shows the proportion of people who fell into each category.

Show that the events of being an adult (A) and wearing spirit gear (W) are dependent.

	Wearing spirit gear	Not wearing spirit-gear	Total
Student	0.6	0.15	0.75
Adult	0.1	0.15	0.25
Total	0.7	0.3	1.00

7. A group of twenty friends went to a movie. At the concession stand, 5 people only ordered popcorn, 2 people just ordered soda, and 9 people ordered popcorn and soda.



(a) Complete the Venn diagram using the information above.

(b) Based on the diagram, are the events of ordering popcorn and ordering soda independent or dependent? Justify.

8. A survey was done to determine the which subject, science or math, a group of high school students prefer. The results are shown in the table below. Is the preference for the subject independent of the grade of the student? Explain how you determined your answer.

	Science	Math
10th Grade	60	40
11th Grade	30	70
12th Grade	20	80

Name: ____________________________________ Date: __________________

MULTIPLYING PROBABILITIES
COMMON CORE ALGEBRA II

Probabilities involving **single-stage experiments** are easy enough because only one thing is happening to affect the probability, i.e. you flip a coin once, you pick one person at random, or you pull one card out of a deck. Probabilities, both empirical and theoretical, become increasingly more complicated with **multi-stage experiments**, where more than one thing happens, i.e. you flip a coin three times. How we handle these types of probabilities actually comes from the conditional probability formula.

***Exercise* #1:** Given that the probability of event B occurring given event A has occurred is $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$ answer the following.

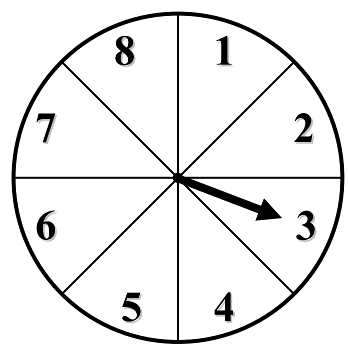
(a) Rewrite this formula, solving for $P(\text{A and B})$.

(b) How could you write this formula if events A and B were **independent**?

This rearrangement of the conditional probability formula gives us a useful tool for calculating the probability of events that occur in **multi-stage experiments**. You will easily be able to accomplish this if you systematically phrase the questions as **intersections of events** (events connected by **AND**).

***Exercise* #2:** Consider the spinner shown below. The spinner is spun twice and the result is recorded.

(a) Are the outcomes of the two spins dependent or independent?



(b) What is the probability that you will get an even on the first spin and a number greater than five on the second spin?

(c) What is the probability that you will spin a prime number and a perfect square (in either order)? Note that this is more complex than (b).

As experiments grow more complicated with more stages, theoretical probability becomes increasingly more complicated. It is especially important to note whether you are sampling **with or without replacement**.

***Exercise* #3:** A class consists of 12 girls and 8 boys. A group of three is picked to give a speech. If the students are picked at random, what is the probability that they all will be boys? Use the events below to show how you calculated your final answer.

Let: E_1 = Event that the first picked was a boy
E_2 = Event that the second picked was a boy
E_3 = Event that the third picked was a boy

The **multiplication property of probability** is crucial in many applications in engineering decision making.

***Exercise* #4:** Say that a power generating facility has three primary safety switches in case of an emergency. The probability that any one of these switches would fail is 5%. What is the probability all three will fail given that the switches are **independent** of one another?

Many times when using the multiplication rule we need to be careful about how we frame the question. But, if we properly frame it in terms of AND and OR logical connectors, then the rules of probability will work out.

***Exercise* #5:** A company was determining the effectiveness of its warranty sales on computers. They took data on the number of customers who purchased warranties on two different brands of computers. If a customer was chosen at random, what is the probability they did not purchase a warranty?

	Percentage of Customers Purchasing	Percent of Those Who Purchased that Also Purchased Warranty
Type 1	68%	35%
Type 2	32%	56%

Name: ____________________________________ Date: __________________

MULTIPLYING PROBABILITIES
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. A fair coin is flipped four times. Find:

   (a) The probability it will land up heads each time.

   (b) The probability it will land the same way each time (slightly different from (a)).

2. A first grade class of nine girls and seven boys walks into class in alphabetical order (by last name). What is the probability that three girls are the first to enter the room? Show your calculation.

   (1) 0.15 (3) 0.35

   (2) 0.20 (4) 0.45

   ____

3. A bag of marbles contains 12 red marbles, 8 blue marbles, and 5 green marbles. If three marbles are pulled out, find each of the following probabilities. In each we specify either replacement (the marbles go back into the bag after each pull) or no replacement.

   (a) Find the probability of pulling three green marbles out with replacement.

   (b) Find the probability of pulling out 3 red marbles without replacement.

   (c) Find the probability of pulling out 3 marbles of the same color without replacement. This is more complex than the other two.

   (d) Find the probability of pulling out two blue marbles and one green marble in any order with replacement. Be careful as there are multiple ways this can be done that will add.

4. The table below shows the percents of graduating seniors who are going to college, broken down into subgroups by gender. If a student was picked at random find the probability that:

	Percent of Graduating Seniors	Percent of Subgroup Going to College
Male	46%	78%
Female	54%	84%

(a) They would be a female going to college.

(b) They would be a male not going to college.

(c) They would be going to college.

(d) They would not be going to college.

5. If a safety switch has a 1 in 10 chance of failing, how many switches would a company want to install in order to have only a 1 in one million chance of them all failing at the same time? Show your reasoning.

REASONING

6. If the probability of winning a carnival game was $\frac{2}{5}$ and Max played it five times, write an expression that would calculate the probability he won the first three games and lost the last two. Use exponents to express your final answer, but do not evaluate.

Name: ____________________________________ Date: _________________

Practice Using Products to Calculate Probabilities
Common Core Algebra II

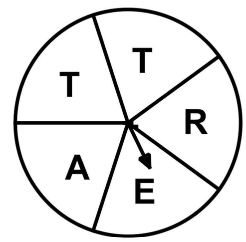
1. Seven vehicles, 5 SUVs and 2 vans, are in line at a drive-thru restaurant. What is the probability that the first four vehicles in the line are all SUVs?

(1) $\frac{120}{2401}$ (3) $\frac{1}{7}$

(2) $\frac{32}{105}$ (4) $\frac{4}{5}$

____

2. The spinner shown below is made up of the letters of the word TREAT. It is spun twice and the results are recorded. Find the probability of getting one vowel and the letter R.



3. A quiz contains 6 multiple choice questions, each with 4 possible answers: A, B, C. and D. Write an expression that would calculate the probability a student gets the first two questions correct and the remaining questions incorrect. Use exponents to express your final answer, but do not evaluate.

4. In a group of 50 students, 36% of the students are seniors and the remaining are juniors. If two different students are chosen to win a prize, determine the probability that they are both juniors. Express your answer as a fraction in lowest terms.

5. The manager of a coffee shop looked at past orders from the store's breakfast hours. She collected data on the orders that included coffee, broken down into whether the customer ordered regular or decaf. Then, she looked at what percentage of each group (regular or decaf) also ordered food. If a customer was chosen at random who ordered coffee, what is the probability they ordered decaf coffee but did not order food?

	% of coffee customers ordering this type	% of those who ordered type who ordered food
Regular	86%	40%
Decaf	14%	57%

6. A small package of jellybeans contains 9 orange, 5 red, and 6 yellow jellybeans. If Jarred randomly selects three jellybeans from the package, find the probabilities that he selected:

(a) 3 red jellybeans with replacement

(b) 3 yellow jellybeans without replacement

(c) Find the probability that Jarred selects 3 of the same color jellybeans with replacement.

(d) Find the probability that Jarred selects and eats 1 red and 2 orange jellybeans.

7. The organizers of a race were curious about whether people who signed up for a race actually finished it. They collected data and broke it down into subgroups according to which race, 5K or 10K, the runner registered for. If a runner was picked at random find the probability that:

	Percent of runners	Percent of subgroup who finished the race
5K	59%	87%
10K	41%	74%

(a) They are a 5K runner who finished the race.

(b) They are a 10K runner who didn't finish the race.

(c) Find the probability that a randomly selected runner finished the race.

Name: ___________________________________

Score: ________ / 26

UNIT #12 ASSESSMENT
COMMON CORE ALGEBRA II

Part I Questions

1. Max accidently mixed 56 turnip seeds with 78 cabbage seeds. Since the two seeds look nearly identical, he must now choose one randomly to plant. Which of the following is the probability he will choose a turnip seed?

(1) 0.42 (3) 0.58

(2) 0.56 (4) 0.72

____

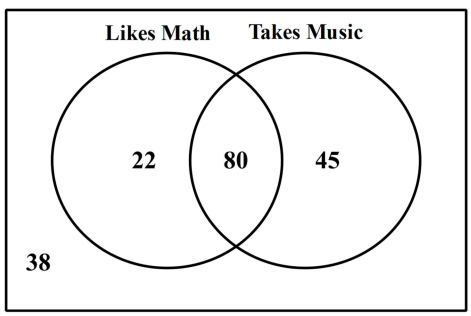
2. A survey was done to investigate the relationship between liking math and learning a musical instrument. The results are shown in the Venn diagram below. If a person was selected at random from this survey, which of the following would be the probability of liking math given that they don't take music?

(1) 0.22

(2) 0.28

(3) 0.33

(4) 0.37



____

3. A school knows that 38% of its incoming students own a smart phone and a tablet. What percent of students who own a smart phone also own a tablet if 84% of students own a smart phone?

(1) 32% (3) 45%

(2) 38% (4) 51%

____

4. If two events, A and B, are independent then which of the following is always true about their probabilities?

(1) $P(\text{A or B}) = P(\text{A}) + P(\text{B})$

(2) $P(\text{A and B}) = \dfrac{P(\text{A})}{P(\text{B})}$

(3) $P(\text{A or B}) = P(\text{A}) \cdot P(\text{B})$

(4) $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$

____

5. In a classroom of 20 students, 70% of them have brown eyes. If a teacher picks two students at random to pass out supplies, what is the probability he chooses two students without brown eyes?

(1) $\frac{49}{100}$ (3) $\frac{3}{38}$

(2) $\frac{91}{190}$ (4) $\frac{9}{100}$

____

6. Over the span of a month, a school tracks student choices at lunch. They find that 56% of all students buy milk with lunch and 32% buy a piece of fruit. If 22% of all students buy both milk and fruit, which of the following represents the percent of students who buy neither?

(1) 28% (3) 42%

(2) 34% (4) 46%

____

7. At a fair, Evie gets five darts she can toss at balloons. If there is a probability of $\frac{1}{10}$ that any given dart she throws will hit a balloon, independent of her other throws, which of the following is closest to the probability she will hit at least one balloon with her five darts?

(1) 41% (3) 54%

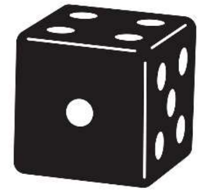
(2) 50% (4) 62%

____

PART II QUESTIONS: Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

8. Jody flips a fair coin three times and each time it lands on heads. Jody believes he can calculate the probability of this occurring by adding the probability of getting a head on each individual flip. Explain why this is incorrect.

9. A fair, standard six-sided die, shown below, is rolled. Explain why rolling a perfect square is independent of whether the roll is even or odd.



PART III QUESTIONS: Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

10. A school is trying to schedule periods of Chemistry and Algebra II. They find that a total of 386 students are taking either one or both of the two courses. If 298 students signed up for Algebra II and 328 have signed up for Chemistry, what would be the probability that a student chosen at random from the 386 will be signed up for both of the courses? Round your answer to the nearest *whole number percent*.

11. In a recent Presidential election, the population that voted was broken down by gender and party as shown in the table below. Given these results, is it more likely that a voter chosen at random voted Republican given that they were male or more likely that they voted Democrat given that they are female? Explain.

	Democrat	Republican
Male	0.22	0.25
Female	0.30	0.23

Name: ___________________________________ Date: _________________

VARIABILITY AND SAMPLING
COMMON CORE ALGEBRA II

Data is everywhere. It's in our newspapers, it's in our science classes, it shows up in economics, medicine and anywhere else that **variability** occurs. Variability is simply the property of **outcomes** being different. The tools of statistics are designed to **explain this variability**.

There are many types of variability. It is good to understand these sources in order to minimize the ones that we are not studying.

***Exercise* #1:** The following types of variability can change uniformity of a data set. For each give an example from any field.

(a) **Observational or Measurement Variability**: Variability that is introduced due to either our measuring instruments not being precise enough or differences in how two different people read the measurement.

(b) **Natural Variability or Inter-Individual Variability**: Variability that accounts for the fact that members of a population are simply different.

(c) **Induced Variability**: This type of variability is in marked contrast to natural. It occurs because we have assigned our population or sample to two or more **treatment** groups and then observe the variability between the groups.

(d) **Sample Variability**: This is the type of variability that occurs when we take multiple **samples** from a **population** randomly. These samples will be different due to the randomness of the sampling process.

Remember, through all of our work in this unit, we are really trying to explain the variability of data within either a population or a sample and then using this to determine if the variability can be attributed to one of the factors above to the exclusion of the others.

There are many different situations in which we collect data. They have important differences and all of them depend on **randomization** in one way or another.

***Exercise* #2:** The three major types of ways to collect data are described below. Give an example of each and explain how **randomization** is part of each method. Randomization is used primarily to eliminate variability caused by some type of **bias**.

(a) **Surveys**: Collections of data from a population where variability is not induced by treatments but by the sample itself (sampling variability).

(b) **Observational Studies**: Collections of data from a population where assignment of individuals from the population into **treatment groups** is **not** under the control of those performing the study.

(c) **Experimental Studies**: In experimental studies individuals are assigned randomly to treatment groups in order to determine the effect of the treatment on the variability of the data. In these cases, the assignment, although random, is under the control of those performing the study.

Random sampling is critical for being able to minimize variability due to **sampling bias**. Random sampling can be done using a variety of different techniques. Simple random sampling can be accomplished using a random number table.

***Exercise* #3:** A list of 10 people's heights, in inches, is shown below.

Person #	1	2	3	4	5	6	7	8	9	10
Height	70	68	60	75	65	69	58	62	66	63

(a) Randomly select five heights from this list by using the random number table that goes with this lesson. Choose a random spot in the table and move down the column. Select the first digit of each number. If you get a repeat, eliminate and keep going. If you get a 0, use this as the 10.

(b) Calculate the **sample mean** to the nearest tenth. Compare to others in the class. What type of variability is being introduced through this process?

Name: ____________________________________ Date: __________________

VARIABILITY AND SAMPLING
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. Scientists randomly select ten groups from a population of men over 50 years old. They calculate the mean weights of each of these groups. The variability between these means can be best attributed to

   (1) measurement variability (3) induced variability

   (2) natural variability (4) sampling variability

   ____

2. Max and Daniel are measuring the amount of time it takes for a ball to roll down a ramp at different heights. For each trial, both Max and Daniel take turns rolling the ball and working the stop watch. They do this in order to quantify which of the following sources of variability?

   (1) measurement variability (3) induced variability

   (2) natural variability (4) sampling variability

   ____

3. Which of the following scenarios would be an attempt to quantify induced variability?

   (1) a phone survey of political preferences during election season
   (2) multiple random samples of products from an assembly line to check for defects
   (3) random assignment of people to a control group and a group taking a drug to lower cholesterol
   (4) recording the variability in the measurement of a soil sample's weight by the same machine

   ____

4. Which of the following research questions would involve collecting data through a survey?

   (1) Watching people exit a grocery store to see the percent who use reusable bags.
   (2) Assigning people to two groups to see the effect of a particular amount of sleep.
   (3) Calling people on the telephone to see if they will be voting in the upcoming election.
   (4) Dropping salt cubes into two different liquids to determine which dissolves faster.

   ____

5. In which of the following cases would an observational study be necessary as compared to an experimental study?

   (1) The study of how increased nutrient levels affect plant growth.
   (2) The study of how educational levels affect median household income.
   (3) The study of how a vaccine affects the percent of mice that get a particular disease.
   (4) The study of how noise level affects the sleep patterns of volunteers in a sleep study.

   ____

6. In an experimental study, a lab wanted to divide volunteers into two groups to determine the effect of a particular phone app to help make people more punctual (on time). The 50 volunteers in the study will be assigned to either a group of 25 who use the app for a week or a group of 25 who do not use the app. The participants were asked to come to a lab to receive the app (or not) at 10:00 am on a Monday. Answer the following questions:

(a) Why would those performing the study *not* want to assign the participants in the two treatments (groups) based on who showed up to the study session first?

(b) Propose a way to use a random number table to generate a simple random selection that eliminates the bias that you discussed in part (a).

7. Two groups of subjects were divided in an experimental study. One group was given a drug to help speed up their metabolism and result in weight loss. The other group was given a **placebo** (a pill that looks identical to the one given to the other group, but without the weight loss drug). After a month of the experiment, the weight loss of each individual in each of the two groups were measured. In general, people in the group who took the metabolism drug did lose more weight, although there were differences in the amount each lost. There are two main types of variability occurring in this study. Describe each type below in the context of this study.

Induced Variability

Natural Variability

REASONING

8. If you were trying to conduct a survey of political preferences for likely voters in an upcoming election and decided to dial 1,000 randomly generated land-line phone numbers (not cell), why might this still introduce **bias** into the sampling?

RANDOM NUMBER TABLE

89679	74452	58378	56038	05793
68479	31125	30744	92830	81733
54958	34875	26881	95459	05001
09728	86396	44698	00445	54666
49645	05086	43332	07908	10593
97742	58396	05140	74052	42483
60394	75922	71275	85120	29034
36606	75808	63047	96796	99834
24656	44208	95016	79816	14185
99387	64057	29448	78761	90544
85213	94939	36368	06737	30994
01727	01497	49402	88141	58513
57535	40645	17498	85894	03705
29613	07446	68202	19465	79334
74042	64704	75418	80166	50113
05561	96960	41774	27701	26791
13709	71189	29285	16286	67827
57752	35321	45784	58222	99383
87272	68090	81526	13161	80735
28664	27875	78093	30888	92618
85995	57330	24519	17238	34929
19402	86361	97351	89230	84306
25335	15291	13878	89663	82143
19631	14030	58249	22092	10967
29731	65359	83185	55700	09254
12342	51338	50542	47077	99987
81333	34849	35289	04468	60304
14825	35419	03873	09164	25581
47865	82527	72916	69732	62153
46246	21019	29652	36296	80016
88454	58304	64450	39653	54792
18412	23667	49507	75752	66366
08044	32980	67699	00755	82771
03017	69707	56600	37524	58097
62259	24785	87969	53877	77589
25294	83064	13116	40659	90535
76449	44295	97098	18216	46682
73964	06143	86782	34176	21466
63960	70532	19083	87598	70803
89628	99681	41047	35674	88642

Name: ___________________________________ Date: _________________

POPULATION PARAMETERS
COMMON CORE ALGEBRA II

When we conduct a study, the complete set of all subjects that share a common characteristic that is being studied is known as the **population**. All populations have **natural or inter-individual variability**. Most of the time, the entire population is not measured, but a sample is taken to infer characteristics of a population. Still, all populations in theory have **population parameters** that describe the population, such as its mean, standard deviation, and interquartile range.

***Exercise* #1:** 18 students in Mr. Weiler's Advanced Calculus class took a quiz with the following results in ascending order.

56, 68, 72, 72, 75, 78, 80, 84, 84, 85, 88, 88, 90, 93, 95, 99, 100, 100

(a) Use your calculator to determine the mean, the median, and the quartiles for this data set. Then, construct a simple box-and-whiskers (box plot) for this data set.

(b) What is the interquartile range of this data set? In theory, what percent of the data set should lie between the first and third quartiles? Is that true for this data set?

(c) What is the population standard deviation for this data set to the nearest tenth? How do you interpret the standard deviation?

(d) What percent of the scores were within one standard deviation of the mean? Within two standard deviations of the mean? Round your percents to the nearest percent and show your work.

Within One Standard Deviation of the Mean

Within Two Standard Deviations of the Mean

Sometimes data is grouped in a frequency chart. We still should be able to calculate the basic population parameters when the information is given in this form.

***Exercise* #2:** A small company has salaries for their 50 employees as given in the table below

Salary (x_i)	Frequency (f_i)
25,000	5
32,000	21
45,000	14
58,000	7
75,000	2
120,000	1

(a) Find the mean and standard deviations of the salary range.

(b) What is the median of this data set? Why is the median considerably lower than the mean in this data set?

(c) Does more or less than 50% of the data set fall within one standard deviation of the mean? Show the analysis that leads to your answer.

Although we have often concentrated on experimental studies where data is collected and means are found, many times we use statistics to represent results of a survey where we are interested in what **proportion** of a **population** share a certain characteristic. These proportions are mostly expressed as decimals, but sometimes are represented by fractions or percents.

***Exercise* #3:** A questionnaire went home to all juniors concerning their ability to bring and use mobile devices at school. The questionnaires constituted a **census** since all of the juniors were surveyed. Of the 742 juniors, 564 of them reported having web-enabled mobile devices. What was the population proportion for web-enabled devices? Express your answer as a decimal and as a percent.

***Exercise* #4:** The proportion of eggs that get cracked in a local egg handling facility is 0.023. If 2,500 dozen eggs are packaged in the factory per day, what should we expect to be the number of eggs cracked per day?

(1) 350 (3) 230

(2) 450 (4) 690

______

Name: ___________________________________ Date: __________________

POPULATION PARAMETERS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following formulas, written in summation notation, would represent the mean of the data set $\{x_1, x_2, ..., x_n\}$? Explain your choice.

(1) $\sum_{i=1}^{n} x_i$ (3) $n\sum_{i=1}^{n} x_i$

(2) $\frac{1}{n}\sum_{i=1}^{n} x_i^2$ (4) $\frac{1}{n}\sum_{i=1}^{n} x_i$ ________

2. The standard deviation of a population characteristics measures

(1) The difference between the maximum and minimum values.
(2) The difference between the third quartile and first quartile values.
(3) The average distance a data value is away from the mean.
(4) The average distance a data value is away from the median. ________

3. The interquartile range of the data set $\{4, 7, 10, 13, 18, 22, 30\}$ is

(1) 15 (3) 7

(2) 18 (4) 10

____

APPLICATIONS

4. If 348 freshmen out of 622 have cell phones, then the population proportion, p, for freshmen cell phone ownership is

(1) 0.56 (3) 0.72

(2) 0.35 (4) 0.44 ________

5. If a population has 824 subjects, then about how many would have characteristics in the upper quartile?

(1) 412 (3) 368

(2) 280 (4) 206

____

6. A school is tracking its freshmen attendance for the first marking period. Shown below is a table summarizing their findings for the 284 members of the freshmen class.

Days Absent (x_i)	Number of Students (f_i)
0	158
1	64
2	18
3	22
4	4
5	7
6	8
9	2
13	1

(a) Find the mean and median number of days absent. Round your mean to the nearest tenth.

(b) What is the population standard deviation for this data set? Round to the nearest tenth.

(c) What proportion of the population that has an absenteeism greater than 4 days?

7. The heights of the 15 players on the Arlington boys' varsity basketball team are given below in inches.

66, 67, 68, 68, 70, 72, 72, 73, 74, 75, 75, 75, 76, 77, 79

(a) Find the mean and standard deviation of this data set. Use the population standard deviation. Round both to the nearest *tenth*.

(b) Determine the proportion of the population that falls within one standard deviation and within two standard deviations of the mean. State your values in decimal form.

One standard deviation from the mean:

Two standard deviations from the mean:

(c) Use the random number table for this lesson to pick a random sample of five players from this list. Do this by picking a random two digit column along the page. Scan down the column until you have picked 5 random integers that fall from 1 to 15. Write down your sample and calculate its mean.

Name: ____________________________________ Date: _________________

THE NORMAL DISTRIBUTION
COMMON CORE ALGEBRA II

Many populations have a distribution that can be well described with what is known as **The Normal Distribution** or the **Bell Curve**. This curve, as seen in the accompanying handout to this lesson, shows the percent or proportion of a normally distributed data set that lies certain amounts from the mean.

***Exercise* #1:** For a population that is normally distributed, find the percentage of the population that lies

(a) within one standard deviation of the mean.

(b) within two standard deviations of the mean.

(c) more than three standard deviations away from the mean.

(d) between one and two standard deviations above the mean.

As can be easily seen from *Exercise* #1, the majority of any normally distributed population will lie within one standard deviation of its mean and the vast majority will lie within two standard deviations. A whole variety of problems can be solved if we know that a population is normally distributed.

***Exercise* #2:** At Arlington High School, 424 juniors recently took the SAT exam. On the math portion of the exam, the mean score was 540 with a standard deviation of 80. If the scores on the exam were normally distributed, answer the following questions.

(a) What percent of the math scores fell between 500 and 660?

(b) How many scores fell between 500 and 660? Round your answer to the nearest whole number.

(c) If Evin scored a 740 on her math exam, what percent of the students who took the exam did better than her?

(d) Approximately how many students did better than Evin?

***Exercise* #3:** The heights of 16 year old teenage boys are normally distributed with a mean of 66 inches and a standard deviation of 3. If Jabari is 72 inches tall, which of the following is closest to his height's percentile rank?

(1) 85^{th} (3) 98^{th}

(2) 67^{th} (4) 93^{rd} ________

***Exercise* #4:** The amount of soda in a standard can is normally distributed with a mean of 12 ounces and a standard deviation of 0.6 ounces. If 250 soda cans were pulled by a company to check volume, how many would be expected to have less than 11.1 ounces in them?

(1) 17 (3) 28

(2) 23 (4) 11 ________

***Exercise* #5:** Biologists are studying the weights of Red King Crabs in the Alaskan waters. They sample 16 crabs and compiled their weights, in pounds, as shown below.

9.8, 10.1, 11.1, 12.4, 11.8, 13.2, 12.8, 12.5, 13.7, 11.6, 13.4, 12.3, 12.6, 14.8, 14.2 15.1

(a) Determine the mean and sample standard deviation for this sample of crabs. Round both statistical measures to the nearest *tenth* of a pound.

(b) Why does this sample indicate that the population would be well modeled using a normal distribution? Explain. Hint – Use your calculator to sort this data in ascending order.

(c) Assuming your mean and standard deviation from part (a) apply to a normally distributed population of crabs caught in Alaska, what percent will fall between 9.6 pounds and 15.6 pounds?

(d) If fishermen must throw back any crab caught below 10.4 pounds, approximately what percent of the crabs caught will need to be thrown back if the weights are normally distributed?

Name: ______________________________________ Date: _________________

THE NORMAL DISTRIBUTION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. A variable is normally distributed with a mean of 16 and a standard deviation of 6. Find the percent of the data set that:

(a) is greater than 16 (b) falls between 10 and 22 (c) is greater than 28

(d) is less than 1 (e) falls between 4 and 19 (f) falls between 22 and 31

APPLICATIONS

2. The weights of Siamese cats are normally distributed with a mean of 6.4 pounds and a standard deviation of 0.8 pounds. If a breeder of Siamese cats has 128 in his care, how many can he expect to have weights between 5.2 and 7.6 pounds?

(1) 106 (3) 98

(2) 49 (4) 111 __________

3. If one quart bottles of apple juice have weights that are normally distributed with a mean of 64 ounces and a standard deviation of 3 ounces, what percent of bottles would be expected to have less than 58 ounces?

(1) 6.7% (3) 0.6%

(2) 15.0% (4) 2.3% __________

4. Historically daily high temperatures in July in Red Hook, New York, are normally distributed with a mean of 84°F and a standard deviation of 4°F. How many of the 31 days of July can a person expect to have temperatures above 90°F?

(1) 6 (3) 9

(2) 2 (4) 4 __________

5. The weights of four year old boys are normally distributed with a mean of 38 pounds and a standard deviation of 4 pounds. Which of the following weights could represent the 90^{th} percentile for the weight of a four year old?

(1) 47 pounds (3) 43 pounds

(2) 45 pounds (4) 41 pounds

______

6. The lengths of songs on the radio are normally distributed with a mean length of 210 seconds. If 38.2% of all songs have lengths between 194 and 226 seconds, then the standard deviation of this distribution is

(1) 16 seconds (3) 8 seconds

(2) 32 seconds (4) 64 seconds

______

7. The heights of professional basketball players are normally distributed with a standard deviation of 5 inches. If only 2.3% of all pro basketball players have heights above 7 foot 5 inches, then which of the following is the mean height of pro basketball players?

(1) 6 feet 5 inches (3) 6 feet 10 inches

(2) 6 feet 2 inches (4) 6 feet 7 inches

______

8. On a recent statewide math test, the raw score average was 56 points with a standard deviation of 18. If the scores were normally distributed and 24,000 students took the test, answer the following questions.

(a) What percent of students scored below a 38 on the test?

(b) How many students scored less than a 38?

(c) If the state would like to scale the test so that a 90% would correspond to a raw score that is one and a half standard deviations above the mean, what raw score is needed for a 90%?

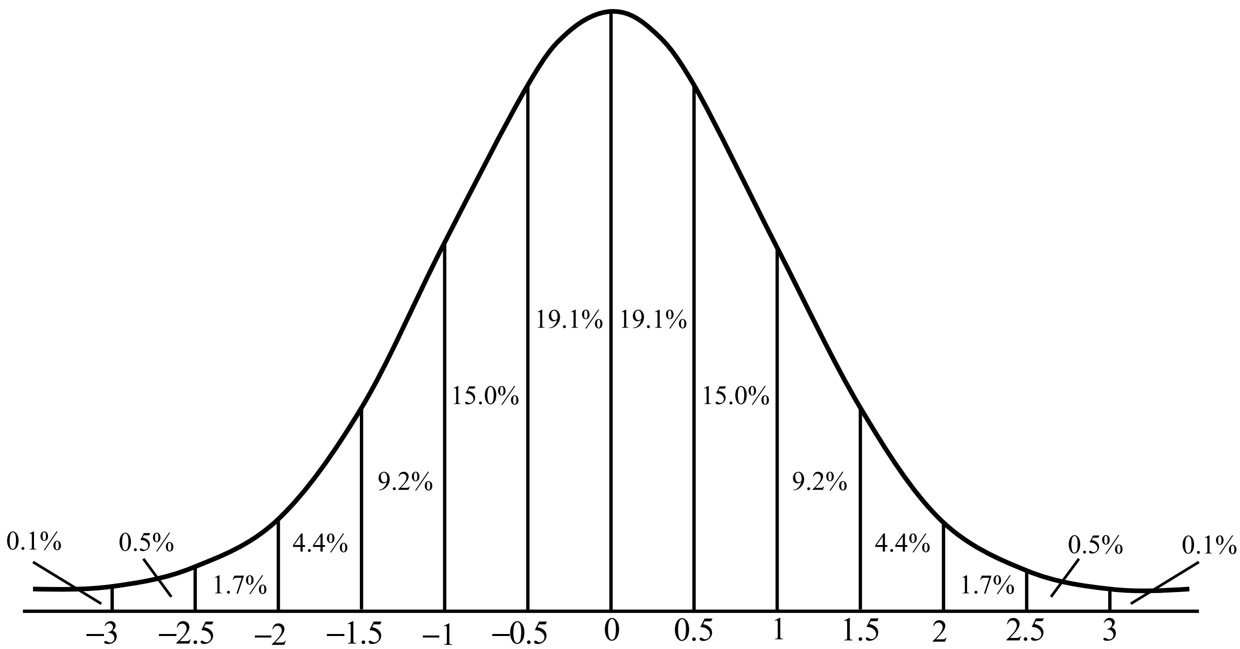
(d) How many of the 24,000 students receive a scaled score greater than a 90%?

(e) The state would like no more than 550 of the 24,000 students to fail the exam. What percent of the total does the 550 represent? Round to the nearest tenth of a percent.

(f) What should the raw passing score be set at so that no more than the 550 students fail?

The Normal Distribution

Based on Standard Deviation



Name: ______________________________ Date: ______________

NORMAL DISTRIBUTIONS– ADDITIONAL PRACTICE
COMMON CORE ALGEBRA II

1. A data set is normally distributed with a mean of 128 and a standard deviation of 8. Show your work that uses the Normal Distribution graph to find the percent of the data set that:

   (a) is greater than 144 (b) falls between 116 and 136 (c) is less than 104

2. A set of heights are normally distributed with a mean of 64 inches and a standard deviation of 3.6 inches. Use your calculator to determine, to the nearest thousandth, the probability that any height chosen at random is:

   (a) greater than 58 inches (b) less than 56.5 inches (c) between 59 and 67 inches

3. A group of 75 teachers were asked about the total number of miles on their daily commute to work. These commute distances were normally distributed with a mean of 36 miles and standard deviation of 8.9 miles. Use the calculator to answer the following questions:

   (a) What percent of the teachers have a daily commute between 18 and 44 miles each day? Round to the nearest tenth of a percent.

   (b) If Marjorie's daily commute is 49 miles, approximately how many teachers have a longer commute than she has?

4. The amount of time students spent designing a poster for their science fair project is normally distributed with a standard deviation of 2.5 hours. If only 0.1% of the students entering the science fair spent less than 1.75 hours, determine the mean number of hours spent on the poster. Express your answer to the nearest hundredth.

5. A population has a mean of $\mu = 39.4$ and a standard deviation of $\sigma = 6.8$. For each of the following data values, calculate the z-score to the nearest hundredth.

   (a) $x_i = 42$ (b) $x_i = 55$ (c) $x_i = 27$

6. The weights of newborn babies at a hospital are normally distributed with a mean of 7.5 pounds and a standard deviation of 0.7 pounds. If Eli weighed 8 pounds 4 ounces when he was born, use the calculator to determine the percentile rank for his weight to the nearest whole number. (Be careful with the units!)

7. A data set is normally distributed with a mean of 729 and a standard deviation of 62. Find the data value, rounded to the nearest whole number, which would result in a z-score of 1.92.

8. A college gave a writing placement test to all incoming students. The mean was 78, the standard deviation was 6.2, and the scores were normally distributed. There were 1500 students who took the placement test. Use your calculator to answer the following questions.

(a) Determine how many students scored between 70 and 85 on the placement test.

(b) A scholarship is offered to students who score in the top 2% of the scores on this test. Determine the minimum score needed to get a writing scholarship. Round to the nearest whole number.

9. Last June, Maria made a list of each day's high temperature $(°F)$ that was recorded in her town in upstate NY. She randomly selected a sample of twenty temperatures from this list.

52, 64, 68, 71, 71, 73, 73, 74, 75, 77, 79, 79, 79, 80, 82, 84, 85, 85, 85, 91

(a) Determine the mean and sample standard deviation for this sample, rounded to the nearest *tenth* of a degree.

(b) Maria wants to use the data from part (a) to answer some questions. Does this sample indicate that the temperatures in this population were well modeled using a normal distribution? Justify your answer.

Name: ____________________________________ Date: __________________

THE NORMAL DISTRIBUTION AND Z-SCORES
COMMON CORE ALGEBRA II

The normal distribution can be used in increments other than half-standard deviations. In fact, we can use either our calculators or tables to determine probabilities (or proportions) for almost any data value within a normally distributed population, as long as we know the population mean, μ, and the population standard deviation, σ. But, first, we will introduce a concept known as a data value's z-score.

THE Z-SCORE OF A DATA VALUE

For a data point x_i, its z-score is calculated by: $z = \frac{x_i - \mu}{\sigma}$. It calculates how far from the mean, in terms of standard deviations, a data point lies. It can be positive if the data point lies above the mean or negative if the data point lies below the mean.

***Exercise* #1:** Boy's heights in seventh grade are normally distributed with a mean height of 62 inches and a standard deviation of 3.2 inches. Find z-scores, rounded to the nearest hundredth, for each of the following heights. Show the calculation that leads to your answer.

(a) $x_i = 66$ inches

(b) $x_i = 57$ inches

(c) $x_i = 70$ inches

Z-scores give us a way to compare how far a data point is away from its mean in terms of standard deviations. We should be able to compute a z-score for a data value and go in the opposite direction.

***Exercise* #2:** Jeremiah took a standardized test where the mean score was a 560 and the standard deviation was 45. If Jeremiah's score resulted in a z-value of 1.84, then what was Jeremiah's score to the nearest whole number?

With z-scores, we can then determine the probability that a subject picked from a normally distributed population would have a characteristic in a certain range. Z-score tables come in many different varieties. The one that comes with this lesson shows only the right hand side, so symmetry will have to be used to determine probabilities.

***Exercise* #3:** The lengths of full grown sockeye salmon are normally distributed with a mean of 29.2 inches and a standard deviation of 2.4 inches.

(a) Find z-scores for sockeye salmon whose lengths are 25 inches to 32 inches. Round to the nearest hundredth.

(b) Use the z-score table to determine the proportion of the sockeye salmon population, to the nearest percent, that lies between 25 inches and 32 inches. Illustrate your work graphically.

***Exercise* #4:** If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75. Answer the following questions by using z-scores and the normal distribution table.

(a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest tenth of a percent.

(b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.

(c) Find the probability that a completed test picked at random would have a score between 500 and 600.

(d) Find the probability that a completed test picked at random would have a score between 600 and 700.

This process is sometimes used to determine a particular data point's **percentile**, which is the **percent of the population less than the data point**.

***Exercise* #5:** The average weight of full grown beef cows is 1470 pounds with a standard deviation of 230 pounds. If the weights are normally distributed, what is the percentile rank of a cow that weighs 1,750 pounds?

(1) 89^{th} (3) 49^{th}

(2) 76^{th} (4) 35^{th}

Your graphing calculator can also find these proportions or percent values. Each calculator's inputs and language will be slightly different, although many will do much of the work for you, even allowing you to **not think about the z-scores**.

***Exercise* #6:** Given that the volume of soda in a 12 ounce bottle from a factory varies normally with a mean of 12.2 ounces and a standard deviation of 0.6 ounces, use your calculator to determine the probability that a bottle chosen at random would have a volume:

(a) Greater than 13 ounces. (b) Less than 11 ounces (c) Between 11.5 and 12.5 ounces

Name: ______________________________ Date: ________________

THE NORMAL DISTRIBUTION AND Z-SCORES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. A population has a mean of $\mu = 24.8$ and a standard deviation of $\sigma = 4.2$. For each of the following data values, calculate the z-value to the nearest hundredth. You do *not* need to read the Normal table.

   (a) $x_i = 30$ (b) $x_i = 35$ (c) $x_i = 19$

   (d) $x_i = 15.4$ (e) $x_i = 24.8$ (f) $x_i = 33.2$

2. A population has a mean of 102.8 and a standard deviation of 15.4. If a data point has a z-value of 1.87 then which of the following is the value of the data point?

   (1) 28.8   (3) 131.6

   (2) 86.7   (4) 152.3

   ____

APPLICATIONS

Get practice with both the Normal Distribution Table and your calculator when doing the following problems.

3. A recent study found that the mean amount spent by individuals on a music service website was normally distributed with a mean of $384 with a standard deviation of $48. Which of the following gives the proportion of the individuals that spend more than $400?

   (1) 0.43   (3) 0.12

   (2) 0.74   (4) 0.37

   ____

4. The hold time experienced by people calling a government agency was found to be normally distributed with a mean of 12.4 minutes and a standard deviation of 4.3 minutes. Which percent below represents the percent of calls answered in less than 5 minutes?

   (1) 4.3%   (3) 6.8%

   (2) 5.3%   (4) 12.9%

   ____

5. The national average price per gallon for gasoline is normally distributed with a mean (currently) of $2.34 per gallon with a standard deviation of $0.26 per gallon. Which of the following represents the proportion of the gas prices that lie between $2.00 and $3.00?

(1) 56%   (3) 84%

(2) 72%   (4) 90%

____

6. If the average teacher salary in the United States is $45,753 and salaries are normally distributed with a standard deviation of $7890, would a salary of $40,000 per year be in the lowest quintile of teacher salary? (Do a quick Internet search on the term quintile if you don't know what it means).

7. The average rent for a one bedroom apartment (in the Winter of 2015) in New York City is a whopping $2801 per month with a standard deviation of $920.

(a) If rents are normally distributed, what percent of the apartments will be less than $2,500 per month?

(b) If rents are normally distributed, how realistic is it to believe you will be able to rent a one-bedroom in New York City for less than $1,500 per month? Justify your answer.

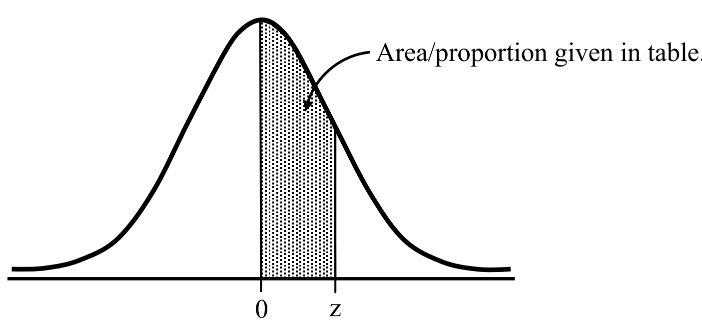
(c) A one-bedroom on the Upper East Side with a doorman and views of Central Park was listed at $5,000 per month. How rare is this? Assume the rents are normally distributed.

(d) Do you think the rents are normally distributed? Keep in mind the normal distribution is symmetric about its mean (looks the same on both sides). If it isn't symmetric, what does it look like?

8. A national math competition advances to the second round only the top 5% of all participants based on scores from a first round exam. Their scores are normally distributed with a mean of 76.2 and a standard deviation of 17.1. What score, to the nearest whole number, would be necessary to make it to the second round? To start, look at the table and see if you can determine the z-value that corresponds to the top 5%.

STANDARD NORMAL DISTRIBUTION BASED ON Z-VALUE

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998



Name: ___________________________________ Date: ___________________

SAMPLING A POPULATION
COMMON CORE ALGEBRA II

The purpose of inferential statistics is to be able to learn something about a population by finding statistics about a sample from the population.

***Exercise* #1:** Why does it make sense to take a sample of a population instead of using the entire population to calculate statistics?

***Exercise* #2:** A population consisting of 200 adult males and females (100 of each) have heights in inches shown in the table and on the dot plot. The population mean is 67.0 inches and the population standard deviation is 6.3 inches (both rounded to the nearest tenth).

(a) This population has two peak heights, one at 65 inches and one at 70 inches. Statisticians would say that this data set is **bimodal**. Why do you think this population is **bimodal**?

(b) Why would a normal distribution **not** fit this population well?

***Exercise* #3:** Take a random sample of 40 of these heights. To do this, use a random integer generator on your calculator to generate numbers between 1 and 200 (the data point number in the shaded columns). Then, write down the value that goes with that data point number. You will certainly get repeated height values but if a data point number repeats, pick another one (circle the 40 you choose in the table). Write your sample values below.

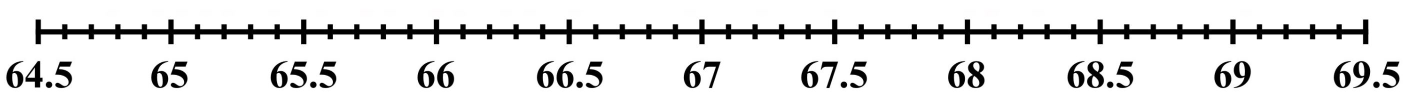
***Exercise* #4:** Calculate the sample mean, $\bar{x}$, and sample standard deviation, s, of your data set. Round both values to the nearest tenth.

***Exercise* #5:** We hope that a sample will fairly represent the overall population. If our sample is **representative** of our population then our sample mean will be close to our population mean and our sample standard deviation will be close to our population standard deviation.

(a) List your sample mean and the population mean below. Was your sample mean an overestimate or underestimate of the population mean?

(b) Do the same for the standard deviation.

***Exercise* #6:** Collect **at least** 20 sample means from fellow classmates (the more the better). Write the sample means below and plot their distribution on the dot plot shown.



Distribution of Sample Means

***Exercise* #7:** Find the mean of the sample means (the average of the averages) from Exercise #6. Find the sample standard deviation of the sample means from Exercise #6. Round both values to the nearest tenth.

***Exercise* #8:** How does the mean of the sample means compare with the population mean? Is it close or far away?

***Exercise* #9:** Is there more variation in the population or in the sample means from Exercise #6? Explain your answer.

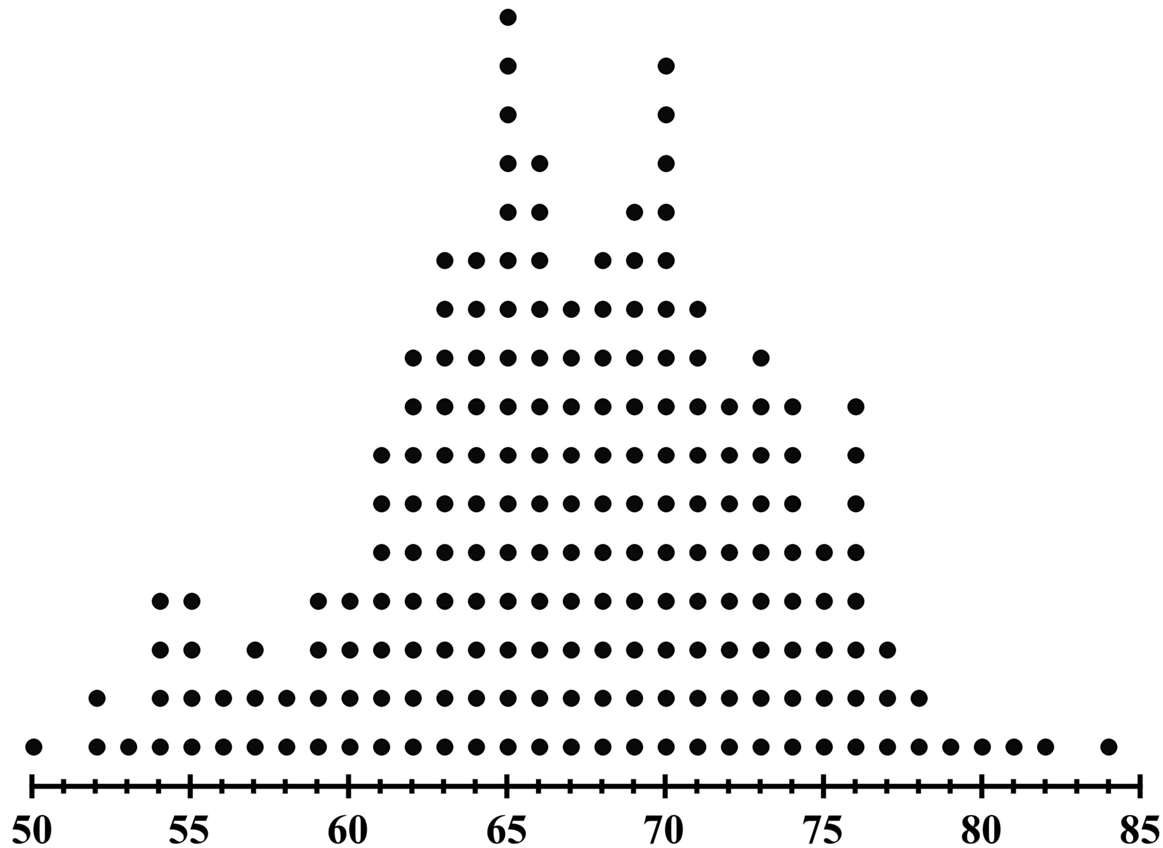
***Exercise* #10:** The **Central Limit Theorem** in statistics states that no matter how the population is distributed, if samples of size n are taken from the population with mean μ and standard deviation σ then the distribution of sample means will have the following characteristics:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

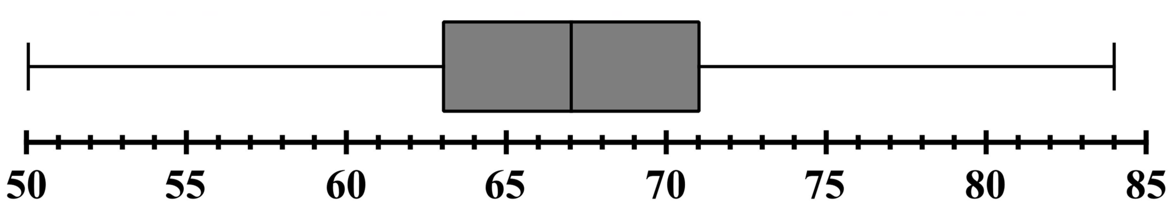
Does your sample of sample means from Exercise #6 support the Central Limit Theorem? Explain.

POPULATION OF 200 HEIGHTS IN INCHES
COMMON CORE ALGEBRA II

Data Point	Value	Data Point	Value	Data Point	Value	Data Point	Value	Data Point	Value
1	70	41	63	81	63	121	74	161	73
2	59	42	59	82	68	122	64	162	73
3	68	43	70	83	66	123	64	163	74
4	65	44	66	84	58	124	73	164	70
5	76	45	62	85	56	125	73	165	65
6	64	46	65	86	61	126	67	166	76
7	62	47	57	87	65	127	78	167	80
8	66	48	61	88	64	128	69	168	66
9	63	49	62	89	66	129	73	169	74
10	65	50	65	90	55	130	78	170	72
11	52	51	63	91	70	131	68	171	69
12	66	52	67	92	60	132	66	172	70
13	70	53	66	93	55	133	64	173	71
14	71	54	63	94	67	134	65	174	79
15	54	55	65	95	59	135	70	175	70
16	62	56	66	96	54	136	70	176	73
17	63	57	63	97	53	137	69	177	75
18	61	58	68	98	73	138	69	178	61
19	60	59	68	99	50	139	84	179	73
20	70	60	69	100	60	140	58	180	72
21	54	61	57	101	72	141	72	181	70
22	68	62	61	102	71	142	67	182	71
23	62	63	56	103	67	143	74	183	76
24	63	64	70	104	71	144	71	184	65
25	57	65	67	105	69	145	75	185	65
26	55	66	73	106	61	146	76	186	66
27	55	67	69	107	72	147	66	187	59
28	68	68	64	108	67	148	60	188	71
29	69	69	76	109	65	149	74	189	66
30	52	70	62	110	66	150	65	190	69
31	65	71	68	111	67	151	61	191	70
32	64	72	69	112	77	152	71	192	69
33	68	73	64	113	68	153	67	193	67
34	62	74	54	114	81	154	65	194	75
35	64	75	63	115	82	155	72	195	77
36	68	76	65	116	76	156	74	196	71
37	62	77	74	117	74	157	70	197	70
38	63	78	63	118	64	158	76	198	75
39	65	79	72	119	77	159	75	199	72
40	62	80	64	120	69	160	71	200	76



Distribution of 200 Heights (inches)



Distribution of 200 Heights (inches)

Population Parameters:

mean = $\mu = 67.0$ in

standard deviation = $\sigma = 6.3$ in

Use the 24 sample means below and your own for questions Exercise #6.

1	**67.625**	**13**	**66.65**
2	**67.325**	**14**	**66.45**
3	**67.275**	**15**	**66.975**
4	**66.65**	**16**	**65.65**
5	**66.625**	**17**	**66.8**
6	**67.00**	**18**	**68.25**
7	**65.55**	**19**	**66.675**
8	**68.15**	**20**	**67.625**
9	**67.1**	**21**	**67.275**
10	**68.225**	**22**	**67.975**
11	**66.05**	**23**	**67.2**
12	**66.2**	**24**	**67.575**

Name: ___________________________________ Date: _________________

SAMPLE MEANS
COMMON CORE ALGEBRA II

The vast majority of the statistics that you've done so far has been **descriptive**. With descriptive statistics, we summarize how a data set "looks" with measures of central tendency, like the mean, and measures of dispersion, like the standard deviation. But, the more powerful branch of statistics is known as **inferential** where we try to **infer** properties about a population from samples that we take. We do this by using **probability** and **sampling variability** to estimate how likely the sample is given a certain population.

We begin our multi-lesson investigation into **inferential statistics** with the most basic question. How can we estimate the **population mean**, μ, if we know a sample mean, $\bar{x}$? Before answering this question, though, we need to investigate the distribution of sample means.

***Exercise* #1:** Say we are investigating the heights of 16 year old American males. Say we know that the population mean height is 65.3 inches with a standard deviation of 4.2 inches. Let's say we take a sample of 16 year old American males. The sample has a size of 30.

(a) Will the mean height of the sample always be 65.3? Why or why not? Could it be significantly different?

(b) Will the standard deviation of the sample be 4.2 inches? Would you expect more or less variation in a sample versus a population?

(c) Run the program NORMSAMP with a mean and standard deviation given above and a sample size of 30. Do at least 200 simulations (500 is preferable). State the minimum and maximum of the sample means, the mean of the sample means, and the standard deviation of the sample means.

min sample mean = _____________ Mean of means: Standard deviation of means:

max sample mean = _____________

(d) How does the **variability** of the sample means compare to the **variability** of the population? Is it more or less?

(e) State the mean of the sample standard deviations. How does it compare to the population standard deviation? Could you use the standard deviation of a sample to estimate the standard deviation of the population?

We can use our simulation to decide whether a sample could have come from a given population. We can even quantify how likely it would be to happen. This is known as establishing **confidence**.

***Exercise* #2:** Mr. Weiler took a sample of 30 16-year old males and found the mean height of the sample to be 66.4 inches. Do you believe this sample came from this population? Why or why not? Examine the results of your simulation. Quantify how likely this sample (or one greater) was to come from the population simulated.

Strangely enough, this process can be used in order to give a **confidence interval** for the population mean if we know the sample mean. This is important because in reality **the population mean is almost never known and is what we want to infer from the sample** mean. The next set of exercises will illustrate how this is done using simulation.

***Exercise* #3:** A sample of 50 ripe oranges were taken from a large orchard in order to estimate the mean weight of a ripe orange. The sample mean was 212 grams and the sample standard deviation was 34 grams.

(a) Why does it seem reasonable to use the sample standard deviation as an estimate of the population standard deviation? See Exercise #1(e).

(b) Run a simulation using the 212 as the population mean (even though it is the sample mean) and use 34 as the population standard deviation. State the 5^{th} percentile sample mean and the 95^{th} percentile sample mean.

5^{th} Percentile Sample Mean = _____________

95^{th} Percentile Sample Mean = ____________

(c) Now, try your simulation again, but use the 5^{th} percentile sample mean as the population mean. Where does the 212 lie on the distribution in terms of percentile? Notice how close this is to the 95^{th} percentile.

(d) Now, try your simulation again, but use the 95^{th} percentile sample mean as the population mean. Where does the 212 lie on the distribution in terms of percentile? Notice how close this is to the 5^{th} percentile.

(e) What both (c) and (d) tell us is that by using the 5^{th} and 95^{th} percentile values based on our original sample mean, we have actually found the **lowest possible population mean** and **highest possible population mean** that could have resulted in that sample mean 90% of the time. Write the 90% confidence interval below for μ based on (b).

Name: ___________________________________ Date: _________________

SAMPLE MEANS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. The mean of the sample means is

   (1) Greater than the population mean.
   (2) Less than the population mean.
   (3) Equal to the population mean.
   (4) Could be greater or less than the population mean. _________

2. The variation within the sample means is

   (1) Less than the variation within the population.
   (2) More than the variation within the population.
   (3) Equal to the variation within the population. _________
   (4) Could be more or less than the variation within the population.

APPLICATIONS

3. A factory has machines that fill 12 ounce soda bottles repeatedly with an average volume of 12.2 ounces and standard deviation of 0.9 ounces. A new machine was installed and 30 bottles were sampled. It was found that they had an average volume of 11.8 ounces. We want to investigate whether this mean is significantly lower than the original population mean.

   (a) Run NORMSAMP with a mean of 12.2 and a standard deviation of 0.9. Run 100 simulations. What percentile rank would you give the 11.8 ounces (this will vary based on the simulation)?

   (b) Based on your findings from (a), can you conclude that this sample mean likely came from the same population or a different population with a lower mean? Explain.

   (c) Use the sample mean of 11.8 ounces and a standard deviation of 0.9 ounces to generate the 90% confidence interval for the population mean of the new soda filling machine by simulation. Use a sample size of 30 and at least 100 samples to generate your interval. Round you lower and upper estimate for μ to the nearest hundredth.

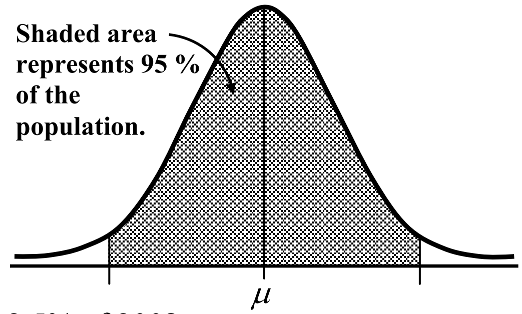
   $\mu_L =$ ___________________

   Interval: __________ $\leq \mu \leq$ __________

   $\mu_H =$ ___________________

Let's work some now with the **95% confidence interval**. It will be easiest to use 200 simulations to generate this interval as we will soon see.

4. Consider a normal distribution with 95% of the probability (or distributions) **centered about the population mean** μ.



(a) What percent lies in half of the shaded area shown in the diagram?

(b) Explain why 2.5% of the population must lie in each of the un-shaded areas of the graph.

(c) What is 2.5% of 200?

5. Researchers have found that the average number of hours per week spent by adults watching television is 34.2 with a standard deviation of 6.8 hours. Researchers wanted to determine if there was an effect to sampling people with tablets. They found that a random sample of 40 people with tablets had a sample mean of 36.3 hours per week with a sample standard deviation of 5.4 hours.

(a) Run NORMSAMP with a population mean of 34.2 hours and a standard deviation of 6.8 hours. Do 200 simulations. This will take some time (approximately 8 minutes). Based on your results, what is the approximate percentile rank of 36.3 (remember it is out of 200 now)?

(b) Do you have significant evidence that the 36.3 comes from a population with a mean that is higher than 34.2? Explain your thinking.

(c) Now, let's attempt to construct the **95% confidence interval** for the sample whose mean was 36.3. Run a simulation with a mean of 36.3, a standard deviation of 5.4, a sample size of 40, and with 200 simulations. Find the 2.5th percentile as the lower limit and the 97.5th percentile as the upper limit.

$\mu_L =$ ____________________

Interval: __________ $\le \mu \le$ __________

$\mu_H =$ ____________________

(d) The **theoretical** (versus simulated) 95% confidence interval can be found using the formula below, where $\bar{x}$ is the observed sample mean and s is the sample standard deviation. Use this formula and compare to the interval from above.

$$\bar{x} - 2 \cdot \frac{s}{\sqrt{n}} \le \mu \le \bar{x} + 2 \cdot \frac{s}{\sqrt{n}}$$

Interval: __________ $\le \mu \le$ __________

Name: ____________________________________ Date: __________________

SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II

Many times we are interested in determining a confidence interval for the population mean, μ, based on a sample mean, $\bar{x}$. Sometimes, though, we want to simply know what proportion, *p*, of a population shares a certain characteristic. We again infer characteristics about *p* based on the proportion of a sample, $\hat{p}$ (p "hat" as it is often called).

***Exercise* #1:** A school is trying to determine the proportion of students who own cell phones. They do a survey of **all** juniors and find that 168 out of 236 of have cell phones. They then take a **sample** of freshmen and find that 30 out of 52 freshmen in the sample own cell phones.

(a) Calculate the population proportion, *p*, of juniors who own cell phones. Round to the nearest hundredth.

(b) Calculate the sample proportion, $\hat{p}$, of freshmen who own cell phones. Round to the nearest hundredth.

Clearly, in the last example, the sample proportion of freshmen who own cell phones is less than the population proportion of juniors who own cell phones. But, can we attribute that variability to the two "treatments", i.e. juniors versus freshmen, or could the variability be due to **sampling variability**, i.e. the random chance that we just picked a group of freshmen who have an unusually low rate of cell phone ownership? We can establish how likely this is to happen by using simulation.

***Exercise* #2:** We would like to determine how likely it is that a sample of 52 out of a population with a proportion of cell phone ownership of 71% or 0.71 would result in a sample proportion of 0.58.

(a) Run the program PSIMUL with a *p* value of 0.71 and a sample size of 52 for 100 simulations. How many of the 100 simulations had a proportion less than or equal to 0.58?

(b) Based on your answer to (a), how likely is it that a sample of 52 from a population with a cell phone ownership of 71% would result in a sample proportion of only 0.58 or less?

(c) Is it possible that a sample of 52 from a population with a cell phone ownership rate of 71% could have a sample proportion of 0.58 or less? Justify your answer.

(d) What conclusion can you make about freshmen cell phone ownership compared to ownership by juniors? Explain.

Inferential statistics is never about proving beyond **any doubt** that a sample either can or cannot come from a certain population. It is about **quantifying how likely it is that it could come from a given population**. Let's continue exploring this question of sample proportions.

***Exercise* #3:** Let's say we have a population with a 0.25 proportion of being 65 years or older. Let's take different sized samples from this population and see how the sample proportions behave. Use the program PSIMUL to simulate a population with a proportion of 0.25 for various sample sizes and 100 simulations.

(a) Fill in the table below.

Sample Size	Low to High $\hat{p}$ values	Range in $\hat{p}$
10		
20		
50		
100		

(b) What was the effect of increasing the sample size on the sample proportions that were simulated? Why does this make sense?

(c) Run PSIMUL one more time with a sample size of 50 but for 200 simulations. Using your results, find the value that represents the 5th percentile of $\hat{p}$ values. Find the result that represents that 95th percentile of the $\hat{p}$ values. Then, write the **90% confidence interval** for this sample size coming from this population.

(d) If researchers surveyed 50 people walking out of a movie and found that 21 of them were 65 years or older, do you believe this sample came from the general population with a $p = 0.25$? Why or why not.

Name: ____________________________ Date: ______________

SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. Historically, the proportion of emperor penguins with adult weights above 60 pounds is 0.64. Take this to be the population proportion for this characteristic.

   (a) A sample of 26 emperor penguins in a zoo found that 20 of the penguins had adult weights above 60 pounds. Calculate the sample proportion, $\hat{p}$, for this sample.

   (b) Run PSIMUL with a population proportion of $p = 0.64$ with a sample size of 26. Do 100 simulations. What percent of these simulations resulted in a $\hat{p}$ value at or above what you found in part (a)?

   (c) Do you have enough evidence from (b) to conclude that penguins raised in a zoo have a significantly higher proportion of weights above 60 pounds? Why or why not?

2. Let's stick with our emperor penguins from #1. Out of a sample of 56 penguins from a zoo, it was found that 43 penguins had weights over 60 pounds. Run PSIMUL again, but now with a sample size of 56. Continue to use $p = 0.64$ and 100 simulations. Do you now have stronger evidence that penguins raised in zoos have a higher proportion with weights over 60 pounds? Explain.

3. In general, as sample size increases, the range in the distribution of sample proportions

   (1) increases (3) decreases

   (2) stays the same (4) could increase or decrease ________

4. In a population with a proportion $p = 0.35$, if samples of size 30 were repeatedly taken, then we would expect approximately 90% of those samples proportions to fall within which of the following ranges?

   (1) 0.28 to 0.42 (3) 0.21 to 0.49

   (2) 0.18 to 0.54 (4) 0.31 to 0.39 ________

5. A sample of the graduating high school class was questioned about their plans after college. We worked with this sample of graduating seniors in our unit on probability. The two-way frequency chart below summarizes the results of the questionnaire. The school would like to investigate the effect of gender on the rate that students go to college.

(a) Calculate the sample proportion of students going to college for the subgroups male and female. Round to the nearest hundredth.

	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

$$\hat{p}(\text{males}) = \frac{\text{number of males going to college}}{\text{total number of males}}$$

$$\hat{p}(\text{females}) = \frac{\text{number of females going to college}}{\text{total number of females}}$$

The proportion for females going to college is higher than for males going to college (a greater percentage of females go to college than males). Is this due to **induced variability** or **sampling variability**? This boils down to asking if the difference is **statistically significant**.

(b) Design a simulation that would test how likely it is for a sample of 30 (the number of men) from a population that has the $\hat{p}(\text{female})$ would result in the $\hat{p}(\text{male})$ **or below**. Explain the simulation and what results you found.

(c) Based on the table above, would you conclude that the overall population proportion of females going to college is greater than the proportion of males? If you believe you have enough evidence from your simulation, explain why. If you do not believe you do, also explain.

(d) Why could this study be an example of both a sample survey and an observational study? Look back at their definitions from the first lesson to fully explain your answer. Also explain how the types of variability introduced.

Name: ________________________________

Score: ________ / 20

UNIT #13 QUIZ (THROUGH LESSON #6)
COMMON CORE ALGEBRA II

1. Mrs. Rose randomly selects 15 equal sized groups of students from a list of all of her current students. For each group, she looks up their scores on the Geometry Regents and then calculates each group's mean score. Which of the following types of variability would most likely explain any differences in these mean scores?

(1) natural (3) sampling

(2) induced (4) measurement

____

2. In a survey of 214 juniors, it was found that 139 of them are taking Algebra II. Which of the following is closest to the proportion of juniors who are *not* taking Algebra II?

(1) 0.35 (3) 0.54

(2) 0.46 (4) 0.65

____

3. A data set is normally distributed with a mean of 26 and a standard deviation of 4. Which of the following is closest to the percent of the data set that is between 22 and 28?

(1) 34.1% (3) 53.2%

(2) 38.2% (4) 68.2%

____

4. The ages of students in a club are normally distributed with a mean of 14 years and a standard deviation of 2.6 years. Which of the following is closest to the z-score for a student who is 17 years old?

(1) 0.50 (3) 0.87

(2) 0.67 (4) 1.15

____

5. Which of the following best describes the mean of the sample means after running a large number of simulations?

(1) It is equal to the population mean.

(2) It is greater than the population mean.

(3) It is less than the population mean.

(4) It may be greater or less than the population mean.

____

6. Lydia and Shannan wanted to explore whether people are more productive while listening to classical music or while working in complete silence. Twenty people were randomly split into two groups and were asked to work on a math puzzle. One group was given noise-cancelling headphones while the other group listened to classical music. After one hour, they collected the puzzles and analyzed the results. Shannan says that they performed an observational study, but Lydia disagrees and says that it was an experimental study. Which girl is correct? Explain your reasoning. [2 points]

7. A group of 50 students took a college placement test to determine which math course they should take during their freshman year. The results are displayed in the frequency table below.

Score	Frequency
60	5
65	10
70	8
75	6
80	6
85	8
90	3
95	2
100	2

(a) Find the mean and the population standard deviation for this set of test scores. Round to the nearest tenth. [2 points]

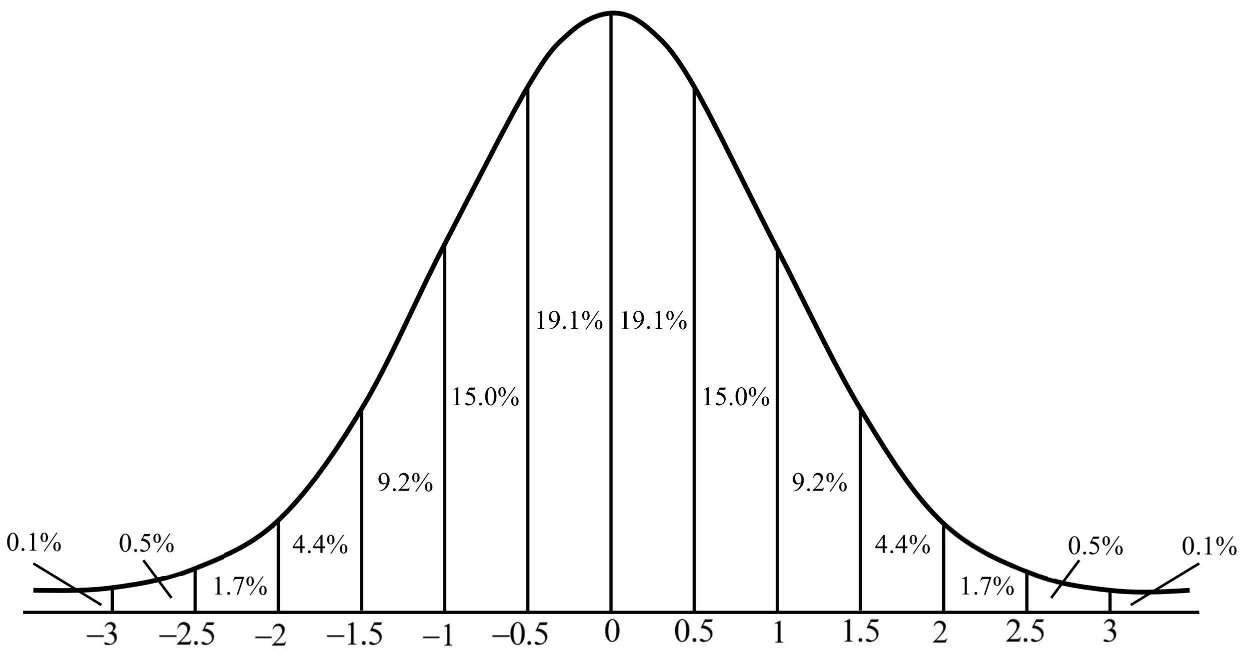
(b) Determine how many scores fall within one standard deviation of the mean. [2 points]

(c) Determine the interquartile range for this set of test scores. [2 points]

8. At a high school, it is known that 76% of all students participate in at least one club. A survey was randomly given to 42 freshmen in their study hall, and 31 students stated that they participate in at least one club. Which is higher, the population proportion of students participating in at least one club or the sample proportion of freshmen who participate in at least one club? Justify your response. [2 points]

THE NORMAL DISTRIBUTION

BASED ON STANDARD DEVIATION



Name: ___________________________________ Date: _________________

THE DIFFERENCE IN SAMPLE MEANS
COMMON CORE ALGEBRA II

In a classic **experiment** two or more **treatment groups** are randomly created and then subjected to the **different treatments**. The question then is how to determine if the variability seen between the two groups is due to the treatment.

***Exercise* #1:** Suppose 50 people were chosen to try out a new diet pill to help increase weight loss. The people are randomly divided into two groups. One is given the pill while the other is given a **placebo** (a pill designed to look the like real one, but with no medicine). There are two main ways variability can be introduced to the results. Discuss each.

Induced (Variability created because of the treatment the subject was placed in):

Natural (Variability just because people, animals, plants, etcetera, are naturally different):

The question, then, is how we can distinguish between the two types. Let's look at a case study.

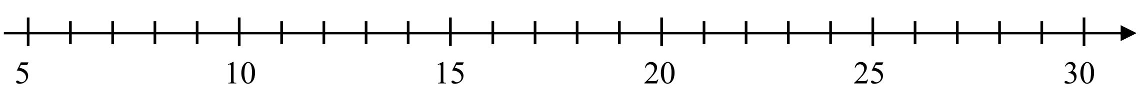
***Exercise* #2:** A seed company is trying to determine the effect of synthetic nutrients versus organic nutrients on the growth rate of corn plants. They select 40 seeds and randomly distribute the seeds to two groups of 20. The seeds in Group 1 are given the organic nutrients and the seeds in Group 2 are given the synthetic nutrients. After three weeks each plant's growth is measured in centimeters.

Group 1 (Organic): 6, 8, 10, 12, 12, 12, 13, 13, 14, 14, 16, 16, 17, 17, 18, 18, 20, 20, 22, 25

Group 2 (Synthetic): 9, 11, 12, 12, 15, 15, 15, 16, 17, 18, 18, 18, 19, 19, 19, 21, 21, 22, 24, 28

Enter these two lists in your calculator. State the mean of each. Then, create a box plot for each using the grid below. You may want to summarize the information you need under each heading.

Group 1 Group 2



***Exercise* #3:** Let's look at the **descriptive statistics** we have so far: the sample means and the box-plot. What does this data suggest? Does it give a clear indication that one treatment resulted in greater plant growth?

There are very sophisticated techniques to probabilistically determine what portion of the variability in these two data sets is due to **natural causes** and what is **induced**. But, we can run a simple simulation which can give us a very good sense. Consider just the question of the **difference in the sample means**. The program MEANCOMP will take our two groups of data and randomly scramble them up into two new groups. It will do that over and over again and calculate the difference in the means of the groups.

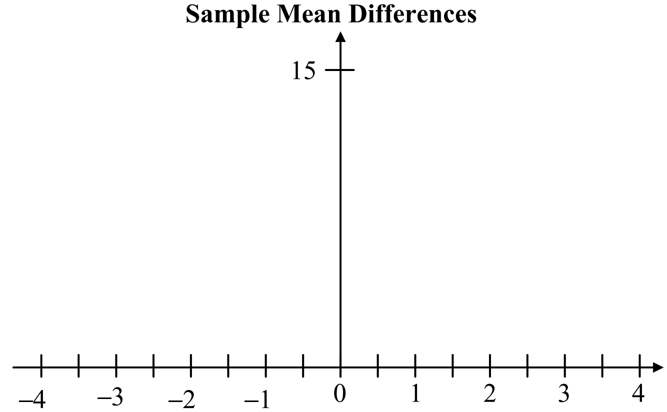
***Exercise* #4:** If $\overline{x_1}$ represents the mean of Group 1 and $\overline{x_2}$ represents the mean of Group 2, do the following.

(a) Find the **observed difference** in the sample means:

$\overline{x_2} - \overline{x_1} =$

(b) Run the program MEANCOMP with 100 simulations. Use your calculator to create a frequency histogram on the axes below for the sample mean differences. Point out where on the histogram the **observed difference** falls.

Sample Mean Differences



(c) Look at the data list containing the sample mean differences. Given there are 100 differences, what percent of the differences were at or above the observed difference?

(d) How confident are you about the **observed difference in sample means** being due to **induced variability** and not **natural variability**? Justify.

Name: ____________________________________ Date: __________________

THE DIFFERENCE IN SAMPLE MEANS
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. In an experiment, two main types of variability are introduced. Explain how both affect the results of the experiment. Give examples to support your descriptions.

   Induced Variability   Natural Variability

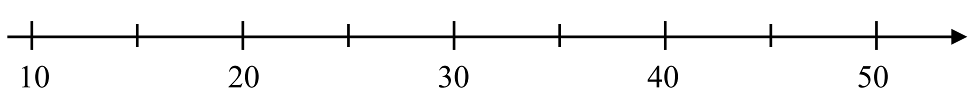
2. A simulator takes the data from the various treatments, randomly scrambles them together to create groups that contain mixed treatments. Explain how this helps quantify the question of natural variability.

3. Researchers in a sleep lab at a college decide to see how a night of no sleep affected the ability of volunteers to answer 50 addition problems in a minute time span. Thirty volunteers were randomly assigned to two groups. Group 1 was not allowed sleep and Group 2 slept normally. Their results, in terms of questions answered out of 50, are given below. As in the lesson, find the sample means and graph a box plot for each.

   Group 1: 11, 14, 16, 17, 23, 25, 25, 27, 30, 31, 33, 34, 34, 36, 38

   Group 2: 18, 22, 24, 25, 30, 30, 32, 33, 34, 34, 36, 37, 42, 44, 48

   Group 1   Group 2



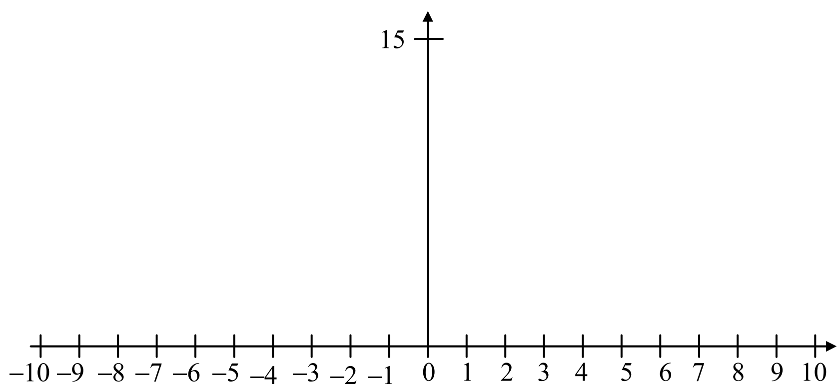
4. Does it appear that getting sleep helps in the ability to answer addition problems? What descriptive statistics can you use to strengthen your argument?

5. Is it true that a person who gets sleep will always answer more addition problems than a person who has not gotten any sleep? Support your answer from the experimental results.

6. Run the program MEANCOMP with 100 simulations on these two data sets. Using your calculator, create a frequency histogram for the sample mean differences on the axes below. Mark on the distribution where the **observed difference** in the sample means lies.

Observed Difference:

$\overline{x}_2 - \overline{x}_1 =$



7. What percent of the simulated differences were greater than or equal to our observed differences? Show your calculation below.

8. Can we **confidently** conclude that the variability in sample means is due to the **treatment** or due to **natural variability**? Support your argument using the distribution above and your answer to 7.

Name: ____________________________________ Date: __________________

THE DISTRIBUTION OF SAMPLE MEANS
COMMON CORE ALGEBRA II

In order to be able to make **inferences** about population parameters based on sample statistics we first must understand how sample statistics, like the sample mean and sample proportion, are distributed. For example, if we take many samples from a population, how will the means of those samples distribute? In this lesson we will investigate this both with simulation and formally.

***Exercise* #1:** Using the Normal Distribution simulator, run a simulation for a population with a mean of $\mu = 50$ and a standard deviation of $\sigma = 15$ for a sample size of 30. Run 100 simulations.

(a) Does the distribution of sample means appear normal (i.e. like a normal distribution)? Explain.

(b) What is the mean of the sample means, symbolized by $\mu_{\bar{x}}$?

(c) What is the standard deviation of the sample means, symbolized by $\sigma_{\bar{x}}$, rounded to the nearest tenth? How does it compare with the standard deviation of the population?

(d) Based on this simulation alone, how likely would it be that a sample of this size taken from this population would have a mean greater than 2 standard deviations, $\sigma_{\bar{x}}$, above the mean?

THE CENTRAL LIMIT THEOREM

When a sample size is fairly large, say 30 or more, then the **distribution of all sample means** of a given size *n* will be **normally distributed** with:

1. **A mean**: $\mu_{\bar{x}} = \mu$ (the mean of the sample means will just be the mean of the population)
2. **A standard deviation**: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$ (the variation of sample means is smaller than that of the population)

***Exercise* #2:** Do the results of your simulation agree with the Central Limit Theorem. Explain.

***Exercise* #3:** The mean height of adult American males is 177 cm with a standard deviation of 7.3 cm. What is the standard deviation of the distribution of samples means from this population with a sample size of 50?

(1) 0.15 (3) 3.54

(2) 1.03 (4) 4.72

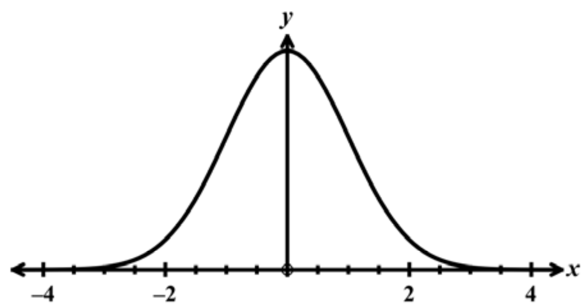
____

Because the distribution of sample means follows a normal distribution, we can determine how likely a sample mean would be given population parameters.

***Exercise* #4:** Jumbo eggs have a mean weight of 71 grams and a standard deviation of 3 grams. A sample of three dozen jumbo eggs was taken at a local egg processing plant and found to have a mean weight of 70 grams. Should this be of concern? Let's explore this question.

(a) What is the standard deviation of sample means of this size from the described population?

(b) What is the z-score for this particular sample mean? Illustrate this on the standard normal curve shown below.



(c) Using either tables or your calculator, determine the probability that a sample of this size would have a mean of 70 grams **or lower**. Round to the nearest tenth of a percent. Shade this area on the normal graph shown in (c).

(d) Why does it make sense in part (c) to determine the probability of having a sample with 70 grams or lower? What does the probability from part (c) tell you?

***Exercise* #5:** Given a population with a mean of 58 and a standard deviation of 12, which of the following represents the probability of getting a sample mean of 61 or greater with a sample size of 50? Show the analysis that leads to your choice.

(1) 7.2% (3) 18.0%

(2) 3.9% (4) 24.2%

____

***Exercise* #6:** In a normal distribution, approximately 95% of all data lie within two standard deviations of the mean. This includes normal distributions of sample means. If a population has a mean of 130, a standard deviation of 8 and samples of size 30 are taken, find the sample mean two standard deviations below the mean and two standard deviations above. Round both means to the nearest hundredth.

Name: ____________________________________ Date: __________________

THE DISTRIBUTION OF SAMPLE MEANS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For each of the population standard deviations given, calculate the standard deviation of the sample means, i.e. $\sigma_{\bar{x}}$, with a sample size of the specified value of n. Show your calculation. Round all answers to the nearest tenth.

(a) $\sigma = 12.5$ and $n = 40$

$\sigma_{\bar{x}} =$

(b) $\sigma = 22$ and $n = 65$

$\sigma_{\bar{x}} =$

(c) $\sigma = 2.7$ and $n = 127$

$\sigma_{\bar{x}} =$

(d) $\sigma = 35$ and $n = 237$

$\sigma_{\bar{x}} =$

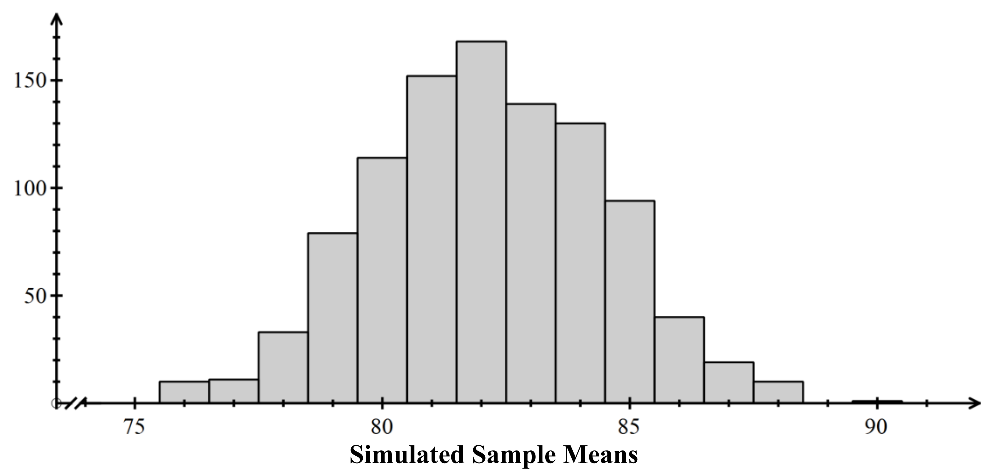
(e) $\sigma = 23.8$ and $n = 30$

$\sigma_{\bar{x}} =$

(f) $\sigma = 18.5$ and $n = 35$

$\sigma_{\bar{x}} =$

2. A simulation is done using a population with a mean of 82, a standard deviation of 15 and samples of size 40. When 1000 samples are simulated, the distribution of simulated sample means is created and shown. The mean of these sample means is 82.09 and the standard deviation is 2.33. Does this simulation support the conclusions of the Central Limit Theorem? Explain.



3. On a standardized test, the standard deviation of the scores was 14.8 points. If samples of size 60 were taken from this test's results, which of the following would be closest to the standard deviation of the means of these samples?

(1) 0.25
(3) 1.91

(2) 0.48
(4) 2.29

____

APPLICATIONS

4. In 2014, new cars had an average fuel efficiency of 27.9 miles per gallons with a standard deviation of 6.8 miles per gallon. A sample of 30 new cars is taken.

(a) What is the probability the sample has a mean gas mileage between 27 and 29 miles per gallon?

(b) What is the probability that the sample has a mean gas mileage greater than 30 miles per gallon?

(c) If a sample of 30 trucks had a sample mean gas mileage of 24.3 miles per gallon, why is it reasonable to assume that all trucks have a lower overall gas mileage than cars? Explain

5. The average length of a cell phone call in 2012 was 1.80 minutes with a standard deviation of 0.32 minutes. A sample of 50 cell phone calls made by users less than 20 years old was taken and had a mean call length of 1.89 minutes.

(a) What is the probability that a sample of 50 from a population with a mean of 1.80 and a standard deviation would have a mean call length of 1.89 minutes or longer.

(b) What conclusion can you make about the average phone call length of users younger than 20 compared to the general population? Explain.

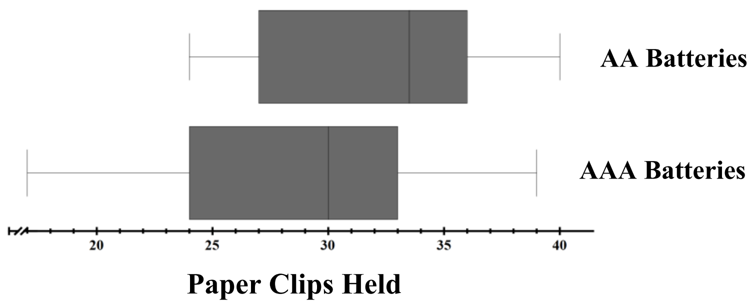
REASONING

6. A population has a standard deviation of 37. If a researcher is designing a study so that the distribution of sample means has a standard deviation of less than 5, what is the smallest sample size that can be used?

Name: ______________________________________ Date: ________________

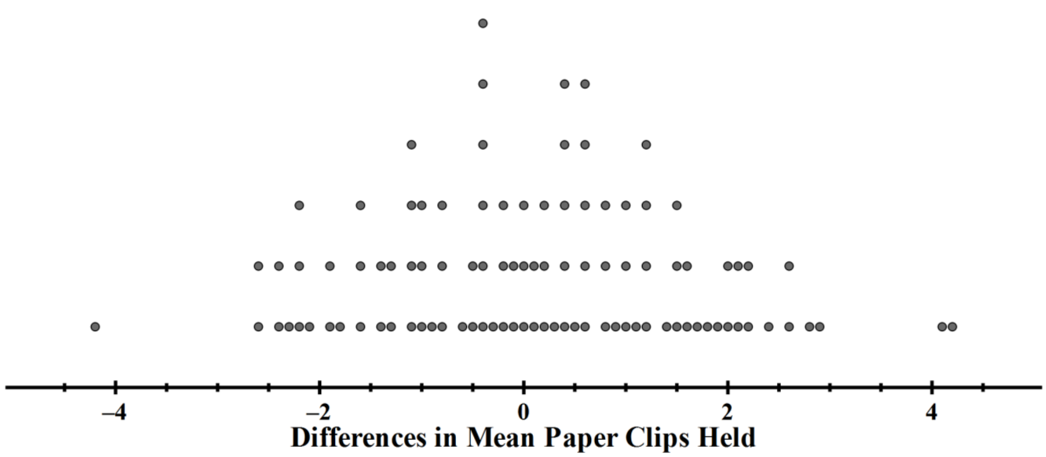
STATISTICAL SIMULATION PACKET
COMMON CORE ALGEBRA II

1. For his science fair project, Max hypothesized that AA batteries create a more powerful electromagnet than AAA batteries. He tested this by hooking up 30 of each type of battery and testing how many paperclips the electromagnet could hold. He found that the 30 AA batteries held on average 3.8 more paper clips than the 30 AAA batteries. The box plot distribution of the two trials is shown below.



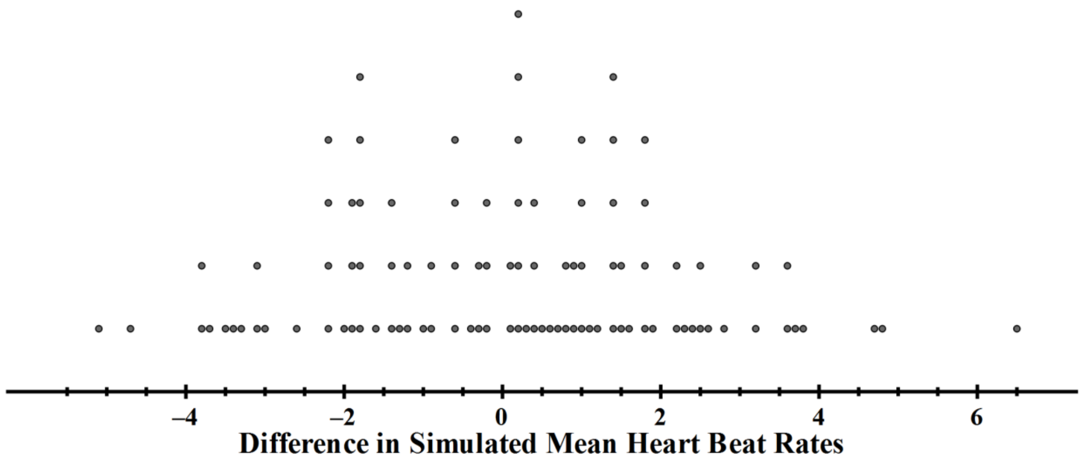
Based on the box plots, explain why it would be incorrect for Max to conclude that AA batteries will always hold more paper clips than AAA batteries.

To determine if the observed difference in the trial means was significant, he randomized the results of the 60 batteries 100 times and calculated the differences in the mean paper clips held for each simulation. The results are shown below.



Explain why, based on these simulations, that Max can conclude that there is statistically significant evidence to support his hypothesis that AA batteries hold more paperclips than AAA batteries.

2. A study was done to determine if regular consumption of coffee had an effect on the resting heart rate of humans. In the study, the resting heart rate of 25 adults who consume coffee and the resting heart rate of 25 adults who don't drink coffee was measured and compared. It was found that the participants in the study who consume coffee had an average heart beat of 3.2 beats per minute faster than those who did not drink coffee. To determine if this difference in means was significant, the 50 data values were randomly assigned to two groups for a total of 100 simulations and the differences in the sample means were calculated. Those results are shown below.



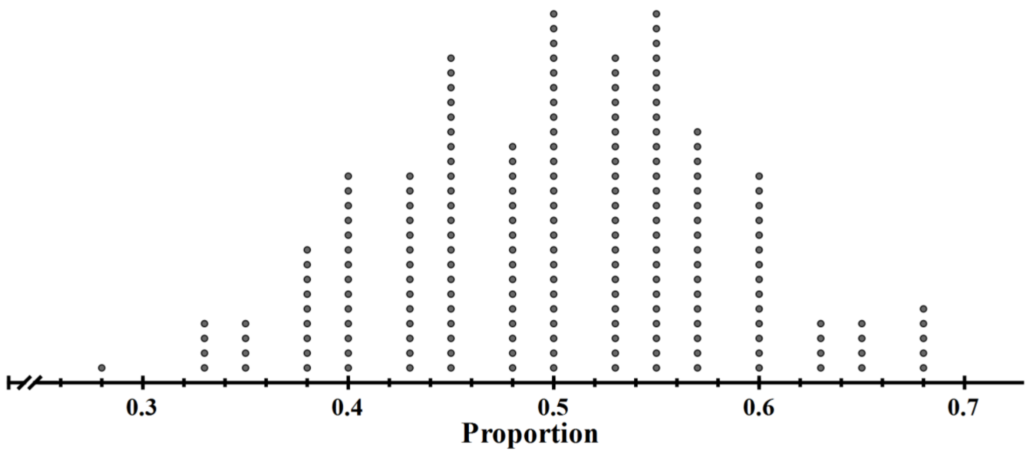
Give an argument for why the observed difference in mean heart rates is not significant based on the simulation results.

Based on the simulation results, give a difference in trial means that would have been significant. Explain your choice.

3. In order to spread news faster, a school would like to start using a common social network app. They will only do so if at least 50% of the student body regularly uses the app. They survey 40 students and find that 18 out of the 40 regularly use the app.

What is the sample proportion for this survey? Show your calculation.

The school still believes there is a reasonable chance that at least 50% of the student body regularly uses the app. They run a simulation of 200 more surveys of 40 students assuming that 50% of the students use the app. The simulation results are shown below.



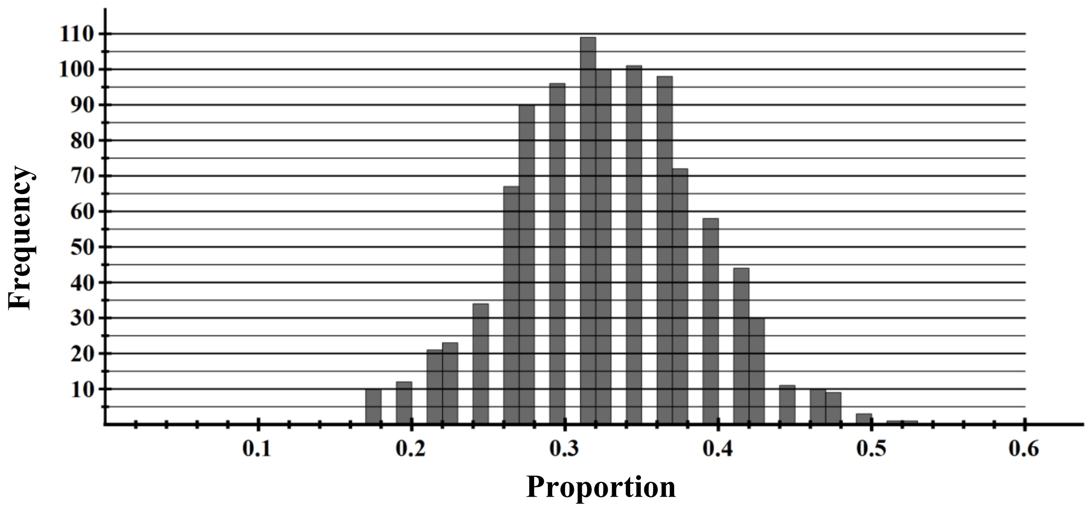
Assuming a 95% confidence level, give an estimate for the margin of error for this simulation. Explain your choice.

Is there evidence from this simulation and the survey that the population uses the app at a rate of at least 50% rate? Explain.

4. The Center for Disease Control and Prevention (CDC) recommends that adults get 7 or more hours of sleep per night. They estimate that 1 in 3 adults do not meet this requirement. A study was done to see if mothers of teenage children were more sleep deprived than the population as a whole. A survey of 60 mothers of teenage children found that 24 out of 60 got less than 7 hours of sleep per night.

What is the sample proportion of sleep deprived mothers of teenage children?

To determine if this sample proportion was significantly different than the population as a whole, researchers ran 1000 simulations with a population proportion of 0.33 and a sample size of 60. The results are shown below.



Explain why the simulation results do not provide statistically significant evidence that mothers of teenage children are more sleep deprived than the adult population on the whole? Explain.

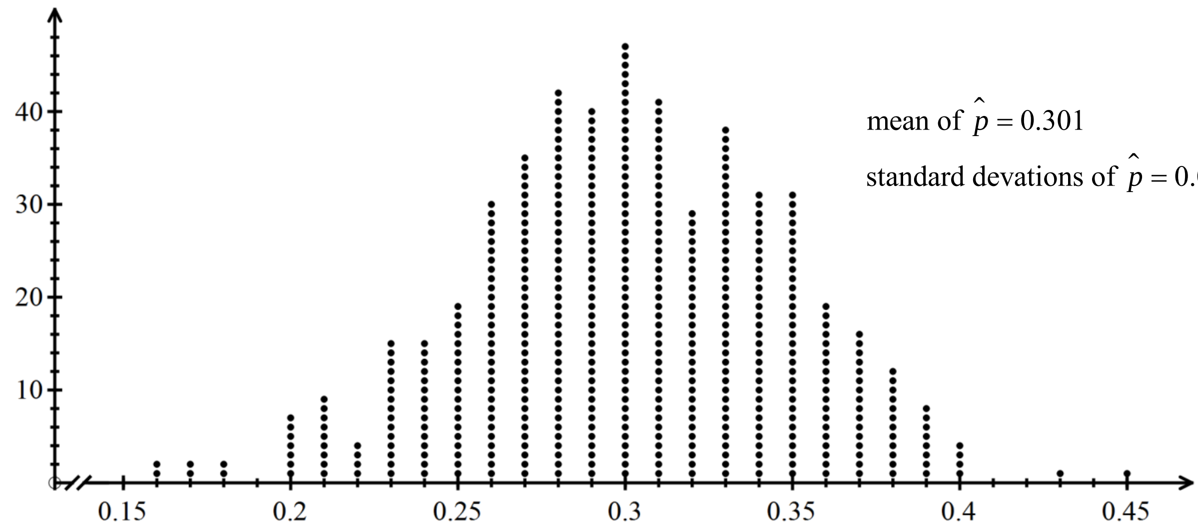
Another survey of 60 fathers of newborns found that 31 of them got less than 7 hours of sleep per night. Do the simulation results now show that fathers of newborns are more sleep deprived than the population as a whole? Explain.

Name: ____________________________________ Date: __________________

THE DISTRIBUTION OF SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II

In the last lesson we saw how the **distribution of sample means** was **normal**. The **Central Limit Theorem** allowed us to find the standard deviation of these sample means. In this lesson, we will look at the same phenomena with sample proportions.

***Exercise* #1:** A simulation of samples taken from a population with a proportion, p, of 0.3 was created. The simulation had a sample size of 100 and 500 simulations were run. The sample proportions, $\hat{p}$, were calculated and their distribution is shown below:



(a) What does the shape of this distribution resemble? Explain.

(b) What is true about the mean of the sample proportions?

The distribution of sample proportions is governed by a very similar phenomena to the distribution of sample means via **The Central Limit Theorem**. The characteristics of the distribution are given below.

THE DISTRIBUTION OF SAMPLE PROPORTIONS

The distribution of sample proportions, $\hat{p}$, from a population with a proportion p and a sample size of n will:

1. Approximate a normal distribution

2. Have a mean of the population proportion, p.

3. Have a standard deviation given by $\sqrt{\dfrac{p(1-p)}{n}}$

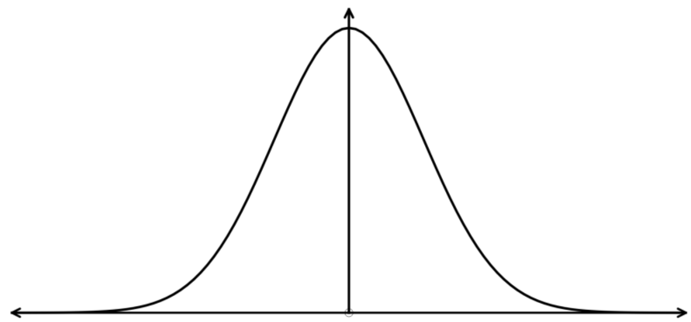
***Exercise* #2:** Does the standard deviation from the simulation agree with that predicted with the above formula?

Since sample proportions will be **normally distributed**, we can perform calculations similar to those done for **sample means**. In other words, we can see how likely a range of sample proportions would be given a particular population proportion.

***Exercise* #3:** Suppose the percent of seniors in high school that own a cell phone is 82%. If a random sample of 50 high school seniors was taken, determine the following:

(a) The standard deviation of sample proportions for this population proportion given this sample size. Show the calculation that leads to your answer.

(b) The probability that the sample proportion will be within 3% of the 82% proportion. Illustrate your work on the general normal curve below.



(c) Find each of the following probabilities. Round each answer to the nearest tenth of a percent.

(i) the sample proportion will be less than 75%

(ii) the sample proportion will be greater than 95%

***Exercise* #4:** Political polls can be tricky. Let's say that 47% of the public will vote for a particular candidate in the upcoming election. If a newspaper takes a random poll of 200 voters, what is the probability that this sample will have a proportion larger than 50%, thus predicting a win for this candidate?

Name: ____________________________________ Date: __________________

THE DISTRIBUTION OF SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For each of the following population proportions, p, find the standard deviation of the sample proportions, $\hat{p}$, given the sample size n. Show your calculation. Round to three decimal place accuracy (nearest thousandth).

(a) $p = 0.34$ and $n = 50$

$\sigma_{\hat{p}} =$

(b) $p = 0.5$ and $n = 400$

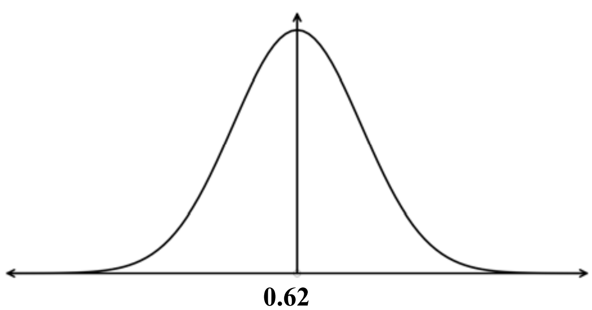
$\sigma_{\hat{p}} =$

(c) $p = 0.25$ and $n = 100$

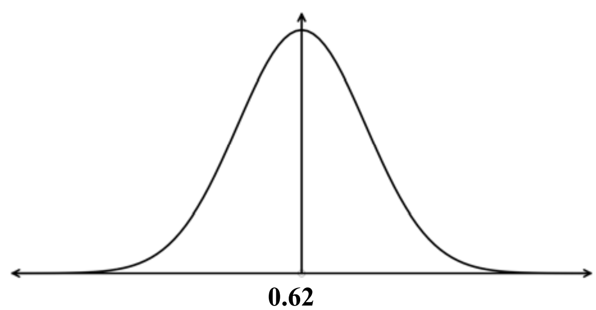
$\sigma_{\hat{p}} =$

2. A population has a proportion of 0.62. A sample of size 40 was taken from this population. Determine the following probabilities. Illustrate each on the normal curve shown below each part.

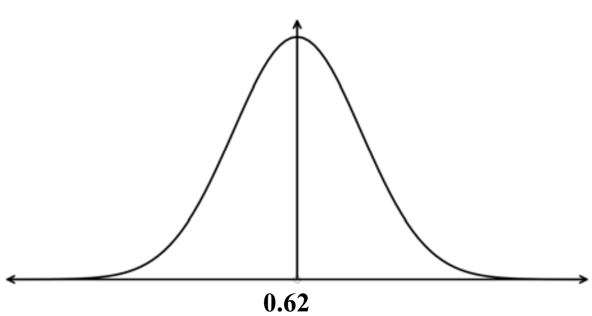
(a) The probability the sample has a proportion between 0.5 and 0.7.



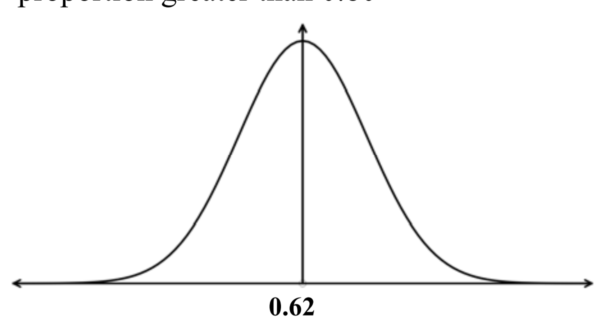
(b) The probability the sample has a proportion within 5% of the population proportion.



(c) The probability that the sample has a proportion less than 0.50.



(d) The probability that the sample has a proportion greater than 0.80



APPLICATIONS

3. A candidate for political office has support from 40% of the public. If a random sample of 100 members of the public was taken, which of the following is closest to the probability that the sample had a proportion of 50% or greater support for this candidate?

(1) 2% (3) 14%

(2) 7% (4) 24%

____

4. A school will offer pizza on Friday's if at least 30% of the students will buy it. A sample of 50 students are asked if they would buy pizza on Friday and 10 respond that they would.

(a) Determine the probability of getting a sample of this size with the proportion or lower given a population with a proportion of 0.30.

(b) Should the school offer pizza on Fridays? Explain your choice by reflecting on what your answer from part (a) tells you.

5. If a 45% of a population likes a particular soda, then what range below shows all sample proportions within two standard deviations of the population proportion if the samples have a size of 70?

(1) 38% to 52% (3) 33% to 57%

(2) 20% to 70% (4) 28% to 62%

____

REASONING

6. Juniors at a high school own internet enabled devices at a rate of 71%. If 52 freshmen were sampled and only 58% of them owned internet enabled devices, is this enough proof to state that freshmen own these devices at a lower rate than juniors? Explain based on probability.

Name: ____________________________________ Date: __________________

MARGIN OF ERROR
COMMON CORE ALGEBRA II

In **inferential statistics** we attempt to **infer** characteristics about a **population** from **sample statistics**. But, these inferences always have a degree of uncertainty in them. Many times, this uncertainty itself is quantified by a **margin of error**, which is a measurement of how accurate we believe our sample statistic to be relative to the population.

***Exercise* #1:** A recent poll found that 36% of all respondents would vote for Candidate A in an election. The poll reported a **margin of error** of 4%. Give an interpretation of what this margin of error means in terms of the 36% support for Candidate A.

The **margin of error** allows us to give a range of values that are reasonable for a population parameter based on a sample statistic. We will only consider what is known as the **95% margin of error**. Although, there are many others based on **confidence level**.

THE 95% MARGIN OF ERROR

The **uncertainty level** that would guarantee a 95% chance that the population parameter falls within a certain range of values.

***Exercise* #2:** In any normal distribution, how much of the data falls within two standard deviations of the mean? Do this quickly using your calculator.

Since roughly 95% of all normally distributed data fall within two standard deviations of the mean, we use two **standard deviations** to develop the **margin of error**. The next exercise will illustrate how this is done for a population proportion.

***Exercise* #3:** In a sample survey, 50 people were randomly sampled about their favorite soda. If 38% of them listed Soda A as their favorite, then answer the following questions.

(a) Based on a proportion of $p = 0.38$, what is the standard deviation of sample proportions of this size, $\sigma_{\hat{p}}$?

(b) What is the margin of error for this survey? What would be an acceptable range of values for the population proportion?

Margins of error are commonly found in surveys and other types of studies that are trying to determine a population proportion based on a sample proportion.

***Exercise* #4:** In a poll of 500 potential voters, Candidate A led Candidate B by a 46% to 39% margin. Could these two candidates actually be tied in the population as a whole? Justify your response.

The **margin of error** can also be useful in working with **sample means**, which will also have a **normal distribution**.

***Exercise* #5:** If a sample of three dozen jumbo eggs had a mean weight of 69.7 grams and a sample standard deviation of 3.2 grams, answer the following question.

(a) What would be a reasonable estimate for the standard deviation of the sample means, i.e. $\sigma_{\bar{x}}$?

(b) Based on (a), what would the margin of error for the population mean weight be?

(c) Jumbo eggs are considered to be eggs with weights at or above 70 grams. Is this within the margin of error for this sample? Explain.

***Exercise* #6:** In 2015, a survey of fifty 20 to 24 year olds was done to determine their mean weekly earnings. The survey found a sample mean of $495 with a standard deviation of $48. If the *World Almanac* reported the 2014 mean weekly earnings of this age range to be $472, do the results of this survey conclusively imply an increase in the mean weekly earning from 2014 to 2015? Explain.

Name: ____________________________________ Date: __________________

MARGIN OF ERROR
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Use the two standard deviation rule to determine the margin of error, to the nearest thousandth, for each of the following proportions with the given sample size. Show the work that leads to your answer.

   (a) $p = 0.35$ and $n = 40$

   (b) $p = 0.72$ and $n = 100$

   (c) $p = 0.5$ and $n = 50$

   (d) $p = 0.25$ and $n = 30$

2. Assuming a population characteristic has a standard deviation, σ, of 38. Calculate the margin of error on the population mean given a sample of each of the sizes given below. Show how you calculate your answer.

   (a) $n = 30$

   (b) $n = 100$

   (c) $n = 1000$

   (d) Generally, as the sample size increases, what happens to the margin of error? Why do you think this occurs?

   (e) What is the minimum sample size needed for the margin of error to be 2 or less? Show or explain how you determined your solution.

APPLICATIONS

3. In an election poll, 200 people were surveyed and 45% expressed their likelihood to vote for a particular candidate. The margin of error on this estimated support is closest to

(1) 2%   (3) 7%

(2) 3%   (4) 12%

4. In a survey of 125 students, 64% of them preferred to start the school day an hour later.

(a) Calculate the margin of error for this survey to the nearest tenth of a percent. Show your work.

(b) The administration of the school will only continue to study the feasibility of starting the day later if there is at least 70% support amongst students. Does this fall within the margin of error of the survey?

5. A consumer group is trying to determine the mean amount that a family of four spends on food per week. They perform a phone survey of 300 random families of four and find a sample mean of \$241.50 with a standard deviation of \$46.72.

(a) Determine an estimate for the standard deviation of the sample means, $\sigma_{\bar{x}}$. Show your calculation.

(b) What is the margin of error for the mean amount spent on food per week?

(c) If the *World Almanac* found that the mean amount spent by all four person families in 2015 was \$244.90, was this within the margin of error you found in (b)? Explain or show how you arrived at your conclusion.

Name: ________________________________ Date: ________________

PRACTICE WITH MARGIN OF ERROR AND 95% CONFIDENCE INTERVALS
COMMON CORE ALGEBRA II

1. Use the two standard deviation rule to determine the margin of error, to the nearest thousandth, for each of the following proportions with the given sample size. Show the work that leads to your answer.

   (a) $p = 0.17$ and $n = 72$

   (b) $p = 0.63$ and $n = 200$

2. A population characteristic has a standard deviation, σ, of 13. Calculate the margin of error on the population mean given a sample of each of the sizes given below. Show how you calculate your answer. Express your answer to the nearest hundredth.

   (a) $n = 60$

   (b) $n = 415$

3. In a sample survey at a local high school, 75 students were randomly sampled about their favorite teacher. If 64% of them listed Teacher A as their favorite, determine an acceptable range of values, to the nearest hundredth, for the population proportion.

4. In a recent survey of 140 teachers, 43% of them stated that cell phone use should be allowed during class. Which of the following is closest to the margin of error for this statistic?

   (1) 2.5%

   (2) 4.1%

   (3) 8.4%

   (4) 11.1%

   ____

5. At a fundraiser, 100 people were surveyed and 21% expressed their likelihood to donate to a particular charity. Determine the 95% confidence interval for expected donations being made to this particular charity.

6. A population characteristic has a standard deviation, σ, of 42. What is the minimum sample size needed for the margin of error to be 3 or less? Show or explain how you determined your solution.

7. In a survey of 175 employees, 77% of them indicated that they would prefer to have a four-day work week.

   (a) Calculate the margin of error for this survey to the nearest tenth of a percent. Show your work.

   (b) The CEO of the company will only explore the possibility of a four-day work week if there is at least 85% employee support. Should the CEO explore switching to a four-day work week?

8. In a recent school district survey of 250 people, 53% of them reported that they were going to vote for a Candidate Martinelli for the school board president. If the survey was reported to have a margin of error of 3%, explain what this means in terms of the actual support for this Candidate Martinelli.

Name: ___________________________________

Score: ________ / 30

UNIT #13 ASSESSMENT
COMMON CORE ALGEBRA II

Part I Questions

1. On a recent Algebra II test, the mean score was a 74. Mr. Weiler decides to add 6 points to each student's score. Which of the following statistical measures will not change after the addition of the 6 points?

   (1) the mean score (3) the median score

   (2) the first quartile (4) the standard deviation

   ____

2. A group of 40 subjects was divided into two groups with one group given an energy drink and one group given water. The subjects were then given a spelling test and the mean number of words correctly spelled was recorded for each member of the two groups. This experiment was designed this way to quantify which of the following types of variability?

   (1) natural (3) induced

   (2) sampling (4) measurement

   ____

3. The length of pop music songs is normally distributed with mean length of 215 seconds and a standard deviation of 38 seconds. What percent of pop songs have lengths between three and four minutes?

   (1) 48% (3) 61%

   (2) 57% (4) 67%

   ____

4. In which of the following scenarios would an observational study be used instead of a survey or experiment?

   (1) a study to quantify the effect of humidity levels on plant growth

   (2) a study to determine how age effects political preference

   (3) a study to measure the level of support for a ballot referendum

   (4) a study to observe the effects of a medicine designed to lower cholesterol __________

5. A population of emperor penguins has a mean weight of 77 pounds with a standard deviation of 16 pounds. What would be the standard deviation of sample means taken from this population with a sample size of 50?

   (1) 0.32 (3) 2.26

   (2) 1.54 (4) 10.89

   ____

6. In a recent poll of 350 likely voters, 42% of them preferred the incumbent candidate. Which of the following would be closest to the margin of error of this statistic?

(1) 2.6%   (3) 4.2%

(2) 3.7%   (4) 5.3%

____

7. A population of fruit flies has a mean life span of 46 days with a standard deviation of 6.2 days. If a sample of 30 fruit flies is taken from this population, what is the probability it will have a sample mean life span of greater than 48 days?

(1) 3.9%   (3) 28.7%

(2) 5.3%   (4) 36.2%

____

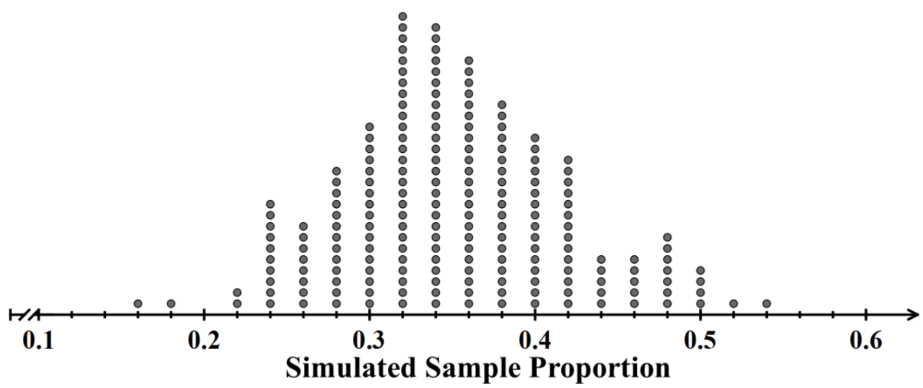
8. A simulation was run 200 times using a population with a proportion of 0.35 and a simulated sample size of 50. The simulated sample proportions are shown below. Based on this distribution, what is the probability of obtaining a sample of 50 with a sample proportion greater than 0.45?

(1) 4.5%

(2) 9%

(3) 14%

(4) 18%



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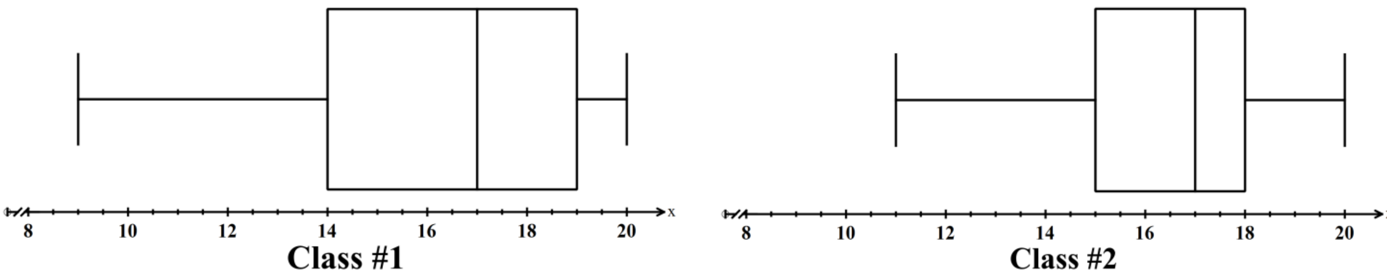
PART II QUESTIONS: Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

9. A 20 point test was given to two classes. Each class had results shown in the box plots below. Which of the two classes has the greater interquartile range?



10. On a standardized test where the results are distributed normally, Elliete scored a 246. If the mean score was a 182 with a standard deviation of 36, then what is Elliete's percentile rank rounded to the nearest whole percent? Explain how you arrived at your answer.

11. A sample of 38 students is chosen from a student body where 58% of all students own cell phones. If 24 of the students in this sample own cell phones, which is higher, the population proportion or the sample proportion?

PART III QUESTIONS: Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

12. Mia believes that corn plants grown with synthetic light will have a slower growth rate than those grown in natural light. She randomly plants 30 seeds, 15 with synthetic light and 15 with natural light. After 4 weeks of growth, she measures each plant's height in centimeters and finds the following:

Treatment #1 - Synthetic Light

$\overline{x_1} = 17.9$ cm

$s_1 = 7.9$ cm

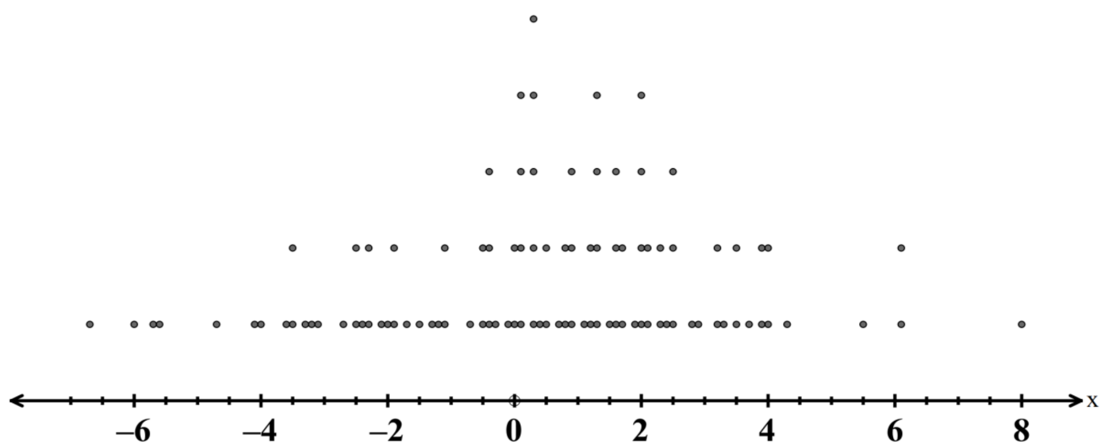
Treatment #2 - Natural Light

$\overline{x_2} = 23.4$ cm

$s_2 = 5.2$ cm

(a) Calculate $\overline{x_2} - \overline{x_1}$. Does this calculation support Mia's belief? Explain. [2 points]

(b) Mia randomly shuffles the 30 results into two groups and repeatedly calculates $\overline{x_2} - \overline{x_1}$ for each simulation. She does this 100 times with the results shown below.



Does the simulation and the results from (a) show a statistically significant increase in plant growth due to the natural light treatment? Explain.